การประยุกต์ใช้ความจุพลวัตรวม ของแผ่นไมโครสตริปรูปวงกลมสำหรับคำนวณ หาความถี่เรโซแนนซ์ของสายอากาศแบบไมโครสตริปบางรูป

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บทคัดย่อ

สูตรเชิงวิเคราะห์อย่างง่ายและถูกต้องของความจุพลวัตรวมของแผ่นไมโครสตริปรูปวงกลม ซึ่งคำนึงถึงผลของการขยายขอบอันเนื่องจากอิทธิพลของสนามขอบที่ขอบของแผ่นได้ถูกนำมาใช้สำหรับกำหนดสภาพยอมพลวัต และขนาดประสิทธิผล และหลังจากนั้นได้นำเอาสูตรทั้งสองนี้ไปใช้ในการคำนวณหาความถี่เรโซแนนซ์ของสายอากาศแบบไมโครสตริปรูปวงกลม รูปสี่เหลี่ยมผืนผ้า รูปสามเหลี่ยมด้านเท่า และรูปวงรี ซึ่งการคำนวณจากวิธีนี้สามารถทำได้อย่างรวดเร็ว ไม่ยุ่งยาก และผลลัพธ์ที่ได้ยังมีความถูกต้องกับข้อมูลในทางการวัด

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Application of the Total Dynamic Capacitance of Circular Microstrip Disk for Calculating the Resonant Frequency of Some Microstrip Antennas

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Abstract

Simple and accurate analytical formula of the total dynamic capacitance of a circular microstrip disk to account for the effect of extended disk due to the influence of the fringing fields at the edge of a circular disk is used for determining the formulae of the dynamic permittivity and the effective dimension. Both formulae are simple and accurate for calculating the resonant frequency of circular, rectangular, equilateral triangular and elliptical disk microstrip antennas. The calculation of the resonant frequency as the function of the dynamic permittivity and of the effective dimension can be done rapidly and the obtained results are in good agreement with the available measured data.

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Introduction

Usually, in microstrip antenna designs, it is important to ascertain the accuracy of the resonant frequency of the antenna because microstrip antennas have narrow bandwidth and can only operate effectively in the vicinity of the resonant frequency. As such, a method to help ascertain the resonant frequency is helpful in antenna designs.

A number of methods have been developed for the determination of the resonant frequency of microstrip antenna [1]-[11]. The numerical methods are time-consuming, while the analytical methods, though less accurate, enable the computation to be done with ease. For engineering applications, less accurate results are often sufficient. These results can be obtained rapidly with simple methods such as cavity model [6], [9]-[11], which suitably describe resonant frequency by a simple design equation. The cavity model also provides an explicit expression for the resonant frequency. To improve calculate frequency from the cavity model, the dynamic permittivity and the effective dimension have been incorporated in the expression to account for the inhomogeneity of medium and fringe field, respectively. The total analytical dynamic capacitance composed with the dynamic main capacitance of Wolff and Knoppik [6] and the static fringing capacitance of Chew and Kong [12] have been attempted to improve the magnetic-wall cavity model. To replace of the relative permittivity ε_r by the dynamic permittivity ε_{dyn}, and of the physical dimension by the effective dimension provide more accurate results for the resonant frequency even for thick substrate.

This paper discusses the physical basis of the Wolff and Knoppik's model [6] and also has been used the static fringing capacitance in [12] for determining the total dynamic capacitance, the effective dimension and the dynamic permittivity of microstrip patches. The present technique studies the effect of effective dimension and uncertainties in the relative permittivity of dielectric substrate on the resonant frequency of some microstrip antennas. The calculated resonant frequencies by using the total dynamic permittivity of circular microstrip disk have been compared against the experimental results and the results obtained from various forms.

Total Dynamic Capacitance and Dynamic Permittivity of Circular Shape

A. Total Dynamic Capacitance

The structures of some microstrip patch antennas are shown in Fig. 1. In the Chew and Kong approximation [12], the total static capacitance of the circular microstrip disk is the summation of the central (main) capacitance of the disk and the fringing capacitance around the boundary (periphery) under the dielectric layer condition. We have assumed that the central capacitance of the disk is not influenced by the fringing field. The assumption has been confirmed with the measurements [10] and [15]-[17] of resonant frequency of the microstrip antennas. Thus, the total dynamic capacitance of the circular microstrip disk is given by

$$C_{t,dyn}(\varepsilon_0\varepsilon_r) = C_{m,dyn} + C_{f,dyn} \qquad (1)$$

where $C_{m,dyn}$ is the dynamic main capacitance (central capacitance) of different modes by ignoring the fringing fields, which depends upon the relative permittivity and the thickness of the dielectric substrate between the disk and ground plane. $C_{f,dyn}$ is the dynamic fringing capacitance around the periphery of disk. Both capacitances can be calculated as follows :

$$C_{m,dyn} = \beta C_{m,stat} \qquad (2)$$

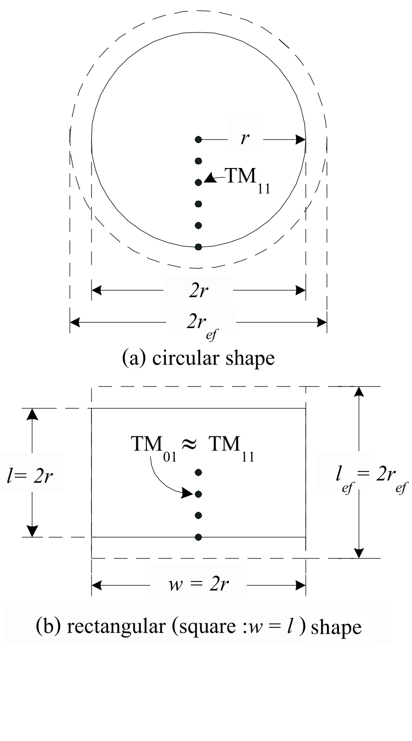
where b is field mode constant depends on the geometrically different shapes of microstrip patch, which for circular shape, the TM_{11} mode, $\beta = 0.3525$ [6] and $C_{m,stat}$ is the static main capacitance given below

$$C_{m,stat} = \frac{\varepsilon_0\varepsilon_r A}{h} \qquad (3)$$

where $A = \pi r^2$ is the area of a circular disk. A dynamic fringing capacitance can be obtained from [6] :

$$C_{f,dyn} = \frac{C_{f,stat}}{\delta_n} \qquad (4)$$

where δ_n is n contant,



(a) circular shape

(b) rectangular (square : $w = l$) shape

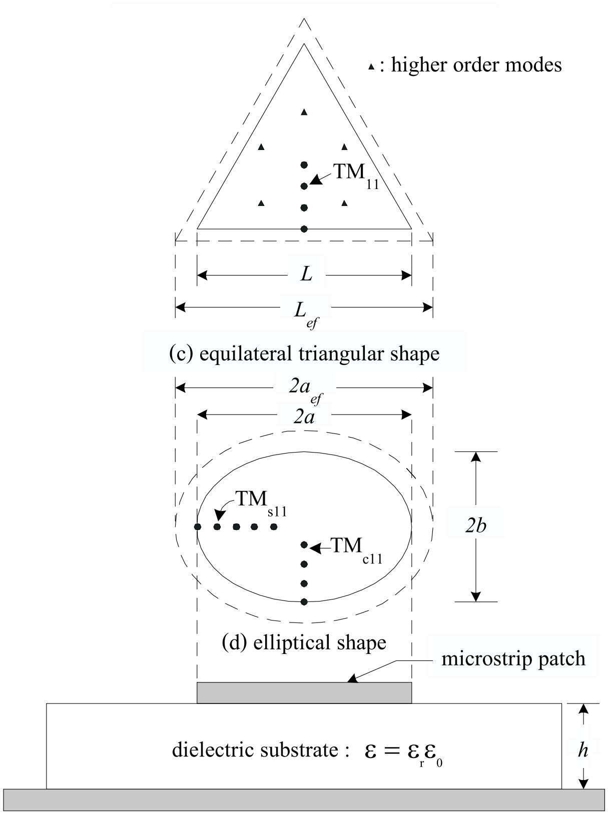


Fig. 1 Geometrically different shapes of microstrip patch.

$$d_n = \begin{cases} 1, \text{ for } n = 0 \\ 2, \text{ for } n \neq 0 \end{cases} \quad (5)$$

with n is half-wave number of electric field intensity distribution in f direction and $C_{f,stat}$ is the static fringing capacitance. As shown in previous works [6], [12], [13], accuracy of the formula for the resonant frequency can be improved with a better approximation for the static fringing (edge) capacitance $C_{f,stat}$, which can be determined from [12] :

$$C_{f,stat} = 2r\varepsilon_0\left[\ln\left(\frac{r}{2h}\right) + (1.41\varepsilon_r + 1.77) + \frac{h}{r}(0.268\varepsilon_r + 1.65)\right] \quad (6)$$

Substituting (2), (3), (4) and (6) into (1) we can obtain a new total dynamic capacitance of a circular microstrip disk for some microstrip antennas given below as :

$$C_{t,dyn}(\varepsilon_0\varepsilon_r) = \frac{\beta\varepsilon_0\varepsilon_r\pi r^2}{h} + \frac{2r\varepsilon_0}{\delta}\left[\ln\left(\frac{r}{2h}\right) + (1.41\varepsilon_r + 1.77) + \frac{h}{r}(0.268\varepsilon_r + 1.65)\right] \quad (7)$$

B. Dynamic Permittivity

The dynamic permittivity ε_{dyn} is a function of the dimensions (w, l and h dimensions), relative permittivity ε_r and the different field mode constant, which can be determined as :

$$\varepsilon_{dyn} = \frac{C_{t,dyn}(\varepsilon_0\varepsilon_r)}{C_{t,dyn}(\varepsilon_0)} \tag{8}$$

where $C_{t,dyn}(\varepsilon_0)$ represents the total dynamic capacitance of a circular microstrip disk in the presence of air, which can be written from (7) as

$$C_{t,dyn}(\varepsilon_0) = \frac{\beta\varepsilon_0\pi r^2}{h} + \frac{2r\varepsilon_0}{\delta}\left[\ln\left(\frac{r}{2h}\right) + 3.18 + \frac{1.918h}{r}\right] \tag{9}$$

The total dynamic capacitance in (7) and the dynamic permittivity in (8) can be used for determining the effective dimensions and the dynamic permittivity of some microstrip patches in the following subsections, respectively.

Circular Microstrip Disk

A. Resonant Frequency

The formula for the resonant frequency of a circular microstrip disk antenna obtained from the cavity model with perfect magnetic wall is given by Wolff and Knoppik [6]. For the present paper, a simple formula [18] was given as

$$f_{11} = \frac{8.78745}{r_{ef}\sqrt{\varepsilon_{dyn}}} \text{ GHz} \tag{10}$$

where f_{11} is the resonant frequency of the dominant mode in GHz, r_{ef} is the effective radius in cm, and ε_{dyn} is the dynamic permittivity (dynamic dielectric constant), taking account to the energy stored in the fringing fields at edge of a circular microstrip disk.

B. Effective Radius

The total dynamic capacitance of the dominant mode (TM_{11} mode) can be obtained from (7) with b = 0.3525 and d = 2 [6] :

$$C_{t,dyn}(\varepsilon_0\varepsilon_r) = \frac{0.3525\varepsilon_0\varepsilon_r\pi r^2}{h} + r\varepsilon_0\left[\ln\left(\frac{r}{2h}\right) + (1.41\varepsilon_r + 1.77) + \frac{h}{r}(0.268\varepsilon_r + 1.65)\right] \tag{11}$$

Equating (11) to equation :

$$C_{t,dyn}(\varepsilon_0\varepsilon_r) = \frac{0.3525\varepsilon_0\varepsilon_r\pi r_{ef}^2}{h} \tag{12}$$

leads to

$$r_{ef} = \left[r^2 + \frac{hr}{0.3525\varepsilon_r \pi} \left\{ \ln\left(\frac{r}{2h}\right) + (1.41\varepsilon_r + 1.77) + \frac{h}{r}(0.268\varepsilon_r + 1.65) \right\} \right]^{\frac{1}{2}} \quad (13)$$

where r_{ef} is the effective radius of a circular disk to account for the fringing fields, which can be used for determining the effective dimensions of some microstrip antennas in the following sections.

C. Dynamic Permittivity

The dynamic permittivity of the dominant mode can be derived from (8) in section II :

$$\varepsilon_{dyn} = \frac{C_{t,dyn}(\varepsilon_0 \varepsilon_r)}{C_{t,dyn}(\varepsilon_0)} \quad (14)$$

where the $C_{t,dyn}(\varepsilon_0 \varepsilon_r)$ and the $C_{t,dyn}(\varepsilon_0)$ in (14) can be defined from the total dynamic capacitance of the circular disk in the presence of dielectric substrate in (11) and the dielectric substrate replaced by air (ε_r= 1) :

$$C_{t,dyn}(\varepsilon_0) = \frac{0.3525\varepsilon_0 \pi r^2}{h} + r\varepsilon_0 \left[\ln\left(\frac{r}{2h}\right) + 3.18 + \frac{1.918h}{r} \right] \quad (15)$$

Rectangular Microstrip Patch

A. Resonant Frequency

The resonant frequency of the TM_{01} mode in the rectangular microstrip patch antenna is given by Wolff [6], which can be determined in a simple formula :

$$f_{01} = \frac{15}{l_{ef}\sqrt{\varepsilon_{dyn}}} \text{ GHz} \quad (16)$$

where f_{01} is the resonant frequency in GHz, l_{ef} is the effective length in cm, and ε_{dyn} is the dynamic permittivity, taking account to the energy stored in the fringing fields at edges of a rectangular microstrip patch.

B. Effective Length

In this paper, to compute the resonant frequency of a rectangular patch, the effect of the fringing fields can be assumed the same as that of an equivalent circular disk, which the effective length l_{ef} of a rectangular in the TM_{01} mode in patch can be approximated by using the effective radius of circular shape in (13) :

$$l_{ef} = 2r_{ef}$$

$$= 2\left\{\left(\frac{l}{2}\right)^2 + \frac{hl}{0.7050\varepsilon_r\pi}\left[\ln\left(\frac{l}{4h}\right) + (1.41\varepsilon_r + 1.77) + \frac{2h}{l}(0.268\varepsilon_r + 1.65)\right]\right\}^{\frac{1}{2}} \quad (17)$$

C. Dynamic Permittivity

The total dynamic permittivity of a rectangular microstrip patch can be calculated from (14), which the $C_{t,dyn}(\varepsilon_0\varepsilon_r)$ and $C_{t,dyn}(\varepsilon_0)$ in (14) can be determined from (11) and (15) as follows :

$$C_{t,dyn}(\varepsilon_0\varepsilon_r) = \frac{0.3525\varepsilon_0\varepsilon_r\pi\left(l/2\right)^2}{h} + \varepsilon_0\, l/2\left[\ln\left(\frac{l}{4h}\right) + (1.41\varepsilon_r + 1.77) + \frac{2h}{l}(0.268\varepsilon_r + 1.65)\right] \quad (18)$$

and

$$C_{t,dyn}(\varepsilon_0) = \frac{0.3525\varepsilon_0\pi\left(l/2\right)^2}{h} + \varepsilon_0\, l/2\left[\ln\left(\frac{l}{4h}\right) + 3.18 + \frac{3.836h}{l}\right] \quad (19)$$

Equilateral Triangular Patch

A. Resonant Frequency

The resonant frequency of the equilateral triangular microstrip patch antenna was given by [14]. In this paper, the resonant frequency formula in [14] is modified to account for the edges extension and the dynamic permittivity due to the fringing fields defined by

$$f_{mn} = \frac{20(m^2 + n^2 + mn)^{\frac{1}{2}}}{L_{ef}\sqrt{\varepsilon_{dyn}}} \text{ GHz} \quad (20)$$

where L_{ef} is the effective length in cm, which will be defined in the following section.

B. Effective Length

As shown in previous work [20], accuracy of the formula for the resonant frequency of the equilateral triangular microstrip patch can be improved with a better approximation by using the total dynamic capacitance of TM_{11} mode of circular disk in (11) as

$$C_{t,dyn}(\varepsilon_0\varepsilon_r) = \frac{0.3525\varepsilon_0\varepsilon_r\pi\left(L/2\right)^2}{h} + \varepsilon_0 L/2\left\{\ln\left(\frac{L}{4h}\right) + (1.41\varepsilon_r + 1.77) + \frac{2h}{L}(0.268\varepsilon_r + 1.65)\right\} \quad (21)$$

The effective length can be derived as

$$L_{ef} = 2r_{ef}$$

$$L_{ef} = 2\left[\left(L/2\right)^2 + \frac{h\left(L/2\right)}{0.3525\varepsilon_r\pi}\left\{\ln\left(\frac{L}{4h}\right) + (1.41\varepsilon_r + 1.77) + \frac{2h}{L}(0.268\varepsilon_r + 1.65)\right\}\right]^{\frac{1}{2}} \quad (22)$$

where L and L_{ef} are the physical and effective length of a equilateral triangular patch, respectively.

C. Dynamic Permittivity

The dynamic permittivity of a equilateral microstrip patch antenna can be defined by using (14), which the $C_{t,dyn}(\varepsilon_0\varepsilon_r)$ in (14) can be obtained from (21). Likewise, $C_{t,dyn}(\varepsilon_0)$ in (14) can be calculated by exchanging ε_r = 1 in (21).

Elliptical Disk

A. Resonant Frequency

An analytic resonant frequency of an elliptical disk microstrip antenna can be derived from the cavity model [22], which is given in a simple formula as

$$f_{c(s)_{mn}} = \frac{15}{ea_{ef}\pi}\sqrt{\frac{q_{c(s)_{mn}}}{\varepsilon_{dyn}}} \text{ GHz} \quad (23)$$

where e is the eccentricity of an elliptical disk :

$$e = \sqrt{1-(b/a)^2} \quad (24)$$

The effective semimajor axis a_{ef} of an elliptical disk in (23) which taking account to the effect of the fringing fields can be approximated from the effective radius of circular disk in (13) as follows :

$$a_{ef} = \left[a^2 + \frac{ha}{0.3525\varepsilon_r\pi}\left\{\ln\left(\frac{a}{2h}\right) + (1.41\varepsilon_r + 1.77) + \frac{h}{a}(0.268\varepsilon_r + 1.65)\right\}\right]^{\frac{1}{2}} \quad (25)$$

Likewise, ε_{dyn} in (23) can be calculated from Section III C.

B. Eigenfunctions $q_{c(s)_{mn}}$

The approximated analytic formula for eigenfunctions $q_{c(s)_{mn}}$ can be derived from [22] and the dominant modes $TM_{c(s)_{11}}$ are used as

$$q_{c11} = -0.0049e + 3.7888e^2 - 0.7228e^3 + 2.2314e^4 \quad (26)$$

and

$$q_{s11} = -0.0063e + 3.8316e^2 - 1.1351e^3 + 5.2229e^4 \quad (27)$$

Comparisons of Measured with Calculated Results

In this section, to verify the calculated resonant frequencies of some microstrip antennas, we present the comparisons of the theoretical resonant frequencies with the measurements of [10] and [15]-[17].

A. Circular Microstrip Disk

The calculated resonant frequencies of circular microstrip disk antennas obtained by using (10) and (11-15) are shown in Table 1, along with the previous theories [6], [7], [15], [18] and measured results [15].

Table 1 Comparisons of Measured and Calculated Resonant Frequencies of Circular Microstrip Disk Antennas

Dimensions (cm)			Resonant frequencies (GHz)					
r	h	ε_r	Measured [15]	Howell [15]	Wolff & Knoppik [6]	Derneryd [7]	Kumprasert & Kiranon [18]	Present Method
3.493	0.1588	2.50	1.570	1.580	1.569	1.537	1.555	1.566
1.270	0.0794	2.59	4.070	4.290	4.267	4.159	4.175	4.208
3.493	0.3175	2.50	1.510	1.580	1.526	1.478	1.522	1.530
13.894	1.2700	2.70	0.378	0.387	0.362	0.350	0.370	0.373
4.950	0.2350	4.55	0.825	0.833	0.836	0.814	0.827	0.824
3.975	0.2350	4.55	1.030	1.037	1.042	1.009	1.027	1.048
2.990	0.2350	4.55	1.360	1.379	1.384	1.332	1.358	1.389
2.000	0.2350	4.55	2.003	2.061	2.067	1.965	2.009	2.057
1.040	0.2350	4.55	3.750	3.963	3.950	3.661	3.744	3.813
0.770	0.2350	4.55	4.945	5.353	5.308	4.848	4.938	4.996

B. Rectangular Microstrip Patch

The calculated resonant frequencies of rectangular microstrip antennas can be compared to the measured resonant frequencies [16] as a check of the validity of the present technique. The theories for the resonant frequency proposed in [2]-[4] and [19] are used for these comparisons. These calculated resonant frequencies obtained by using these methods and available measured results [16] are shown in Table 2.

Table 2 Comparisons of Measured and Calculated Resonant Frequencies of Rectangular Microstrip Patch Antennas

Dimensions (cm) ε_r = 2.33			Resonant frequencies (GHz)					
w	l	h	Measured [16]	James et al. [3]	Hammerstad [2]	Chew & Liu [4]	Kumprasert & Kiranon [19]	Present Method
5.70	3.80	0.3175	2.31	2.30	2.38	2.38	2.37	2.37
4.55	3.05	0.3175	2.89	2.79	2.90	2.91	2.90	2.88
2.95	1.95	0.3175	4.24	4.11	4.34	4.33	4.27	4.25
1.95	1.30	0.3175	5.84	5.70	6.12	6.08	5.94	5.90
1.70	1.10	0.3175	6.80	6.47	7.01	6.91	6.74	6.70
1.40	0.90	0.3175	7.70	7.46	8.19	7.88	7.81	7.77
1.20	0.80	0.3175	8.27	8.13	9.01	8.40	8.51	8.44
1.05	0.70	0.3175	9.14	8.89	9.97	8.70	9.32	9.25
1.70	1.10	0.3175	7.87	7.46	7.84	7.83	7.75	7.71
1.70	1.10	0.3175	6.80	6.47	7.01	6.91	6.74	6.70

C. Equilateral Triangular Patch

The theoretical frequencies of equilateral triangular microstrip patch antenna which have been obtained by using (20) are shown in Table 3. Comparisons of the resonant frequencies among previous calculated results [8]-[10], [20] and the measured results [10] are also given.

Table 3 Comparisons of Measured and Calculated Resonant Frequencies of Equilateral Triangular Microstrip Patch Antennas (L = 10 cm, ε_r = 2.32, and h = 0.159 cm)

Modes	Resonant frequencies (GHz)						
	Measured [10]	Suzuki & Chiba [8]	Singh et al. [9]	Dahele & Lee [10]	Kumprasert & Kiranon [20]	Present Method user ε_r	Present Method user ε_{dyn}
TM_{10}	1.280	1.273	1.273	1.299	1.289	1.280	1.306
TM_{11}	2.242	2.239	2.205	2.252	2.233	2.217	2.261
TM_{20}	2.550	2.546	2.546	2.599	2.579	2.559	2.611
TM_{21}	3.400	3.419	3.369	3.439	3.411	3.386	3.454
TM_{30}	3.824	3.819	3.820	3.899	3.868	3.839	3.917

D. Elliptical Disk Microstrip Antenna

The resonant frequencies of the elliptical disk microstrip antenna are calculated for TM_{c11} and TM_{s11} modes. The calculated and measured results [17] are compared and they are shown in Tables 4 and 5.

Table 4 Measured and Calculated Resonant Frequencies of the Elliptical Disk Microstrip Antenna with a = 4 cm, ε_r = 2.48 and h = 0.1575 cm.

b/a	Resonant frequencies (GHz)							
	Measured [17]		Kumprasert & Kiranon [21] used ε_r		Present Method *used ε_r		Present Method **used ε_{dyn}	
	f_{c11}	f_{s11}	f_{c11}	f_{s11}	f_{c11}	f_{s11}	f_{c11}	f_{s11}
0.962	1.385	1.410	1.411	1.438	1.389	1.417	1.446	1.474
0.976	1.378	1.400	1.406	1.422	1.385	1.401	1.441	1.458
0.983	1.370	1.380	1.404	1.415	1.383	1.394	1.439	1.451
0.996	1.370	1.370	1.401	1.403	1.379	1.382	1.436	1.439

Table 5 Measured and Calculated Resonant Frequencies of the Elliptical Disk Microstrip Antenna with a = 4 cm, ε_r = 2.48 and h = 0.3175 cm.

b/a	Resonant frequencies (GHz)							
	Measured [17]		Kumprasert & Kiranon [21] used ε_r		Present Method *used ε_r		Present Method **used ε_{dyn}	
	f_{c11}	f_{s11}	f_{c11}	f_{s11}	f_{c11}	f_{s11}	f_{c11}	f_{s11}
0.960	1.342	1.360	1.368	1.397	1.333	1.361	1.418	1.448
0.976	1.348	1.348	1.363	1.379	1.328	1.343	1.413	1.429
0.985	1.328	1.328	1.361	1.370	1.325	1.335	1.410	1.420
0.993	1.320	1.320	1.359	1.364	1.324	1.328	1.409	1.413

* dispersion effects of ε_r does not happen (at frequency below ≈ 1.50 to 2.00 GHz) used ε_r.

** dispersion effects of ε_r happen ($\varepsilon_r \longrightarrow \varepsilon_{dyn}$ at frequency higher ≈ 2.00 GHz) used ε_{dyn}.

Conclusion

In this paper, a technique for calculating the resonant frequency of circular, rectangular, equilateral triangular and elliptical disk microstrip antennas by using the approximation of the total dynamic capacitance of a circular microstrip disk is presented. The resonant frequencies calculated by the present technique are closed to the measured results. The present technique is also simple and adequate for engineering applications, which can be applied to different shapes of microstrip patch antennas.

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