วิธีที่แม่นยำขึ้นในการประมาณค่าการประหยัดพลังงาน จากการควบคุมเครื่องสูบแบบหอยโข่งด้วยการปรับความเร็วรอบ

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บทคัดย่อ

วิธีพื้นฐานสำหรับการประมาณค่าการประหยัดพลังงานจากการควบคุมความเร็วรอบของเครื่องสูบแบบหอยโข่งคือ การใช้กฎความคล้ายของเครื่องสูบ (pump affinity laws) ซึ่งง่ายต่อการใช้งาน อย่างไรก็ตาม วิธีนี้อาจทำให้ได้ผลลัพธ์ที่ ไม่ถูกต้องถ้าระบบของเครื่องสูบมีเฮดสถิตหรือเฮดความดัน บทความนี้มีวัตถุประสงค์เพื่อนำเสนอวิธีที่แม่นยำขึ้น วิธีนี้ สร้างแบบจำลองคณิตศาสตร์อย่างง่ายของเครื่องสูบซึ่งมีพารามิเตอร์อิสระของเครื่องสูบและระบบเพียงไม่กี่ตัว แบบ จำลองดังกล่าวใช้สร้างความสัมพันธ์ระหว่างลักษณะเฉพาะไร้มิติกับอัตราการไหลไร้มิติที่ทุกความเร็วรอบของเครื่องสูบ ซึ่งนำไปสู่แผนภูมิการใช้กำลังงานของเครื่องสูบ (pump power consumption chart) การคำนวณการใช้พลังงานของ เครื่องสูบที่ปรับความเร็วรอบได้ด้วยแผนภูมินี้แสดงให้เห็นว่า ผลต่างระหว่างการใช้แผนภูมิกับการใช้กฎความคล้ายเพิ่ม ขึ้นตามเฮดสถิต

A More Accurate Method of Estimating Energy Saving by Variable-speed Control of Centrifugal Pump

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Abstract

The basic method for estimating energy saving by variable-speed control of centrifugal pumps is by using the pump affinity laws, which are easy to use. However, this method is erroneous if the pumping system has static head or pressure head. This paper aims at providing a better method of estimation. This method is based on a simplified mathematical model of centrifugal pumps using the minimum number of pump and system parameters. This model yields dimensionless pump characteristics as functions of dimensionless flow rate at an arbitrary pump speed. It is subsequently used to construct a pump power consumption chart. Computation of power consumption by variable-speed pumps using this chart shows that the difference between results from the power consumption chart and the pump affinity laws increases as the static head of the pumping system increases.

1. Introduction

In many centrifugal pump applications, it is necessary to vary flow rate. Figure 1 shows that the initial operation point of the pump is the intersection between the pump head curve A-A' and the system curve X-X. Therefore, one way to vary the flow rate is to vary the system curve by adjusting the control valve installed in the system. It can be seen from Fig. 1 that this method results in a steeper system curve X-X', and the intersection at a reduced flow rate. It is well known that this method consumes a lot of energy. A more efficient method to reduce flow rate is by changing the pump head curve using a different pump speed without changing the system curve. The intersection between the new pump head curve B-B' and the system curve X-X is at the same reduced flow rate, but lower head, and lower power consumption. The energy efficiency of variable speed control for pumps has led to its installation in many systems [1-4].



Fig. 1 Two methods of changing the flow rate of a pump.

Installing variable speed control for pump incurs considerable expense, which must be justified by energy saving. A convenient way to compute energy saving is by using the affinity laws, which state that

$$\frac{Q_1}{Q_2} = \frac{N_1}{N_2} \tag{1}$$

$$\frac{H_1}{H_2} = \left(\frac{N_1}{N_2}\right)^2$$
(2)

$$\frac{P_1}{P_2} = \left(\frac{N_1}{N_2}\right)^3 \tag{3}$$

where Q is flow rate, N is pump speed, H is head, and P is power. Equations (1) and (3) may be combined into

$$\frac{P_1}{P_2} = \left(\frac{Q_1}{Q_2}\right)^3 \tag{4}$$

which is used to compute P_2 if P_1 , Q_1 , and Q_2 are known. Substantial energy saving may be expected if Eq. (4) is presumed to be true. However, energy saving does not depend on the pump alone; the system plays a significant role in determining how much saving can be realized. In fact, Eq. (4) is valid only when there is no static head in the system. Figure 2 shows that, in presence of static head, the new pump speed N_2 is larger than the value computed from Eq. (1), resulting in higher head and more power required to run the pump. Relying on Eq. (4) to calculate energy saving by variable speed control may, therefore, lead to the underestimation of the payback period required for the investment in variable speed control.



Fig. 2 Demonstration that more power is required to run the pump if there is static head in the system.

Although there have been several attempts to reduce errors caused by the affinity laws [5-8], it is suggested in this paper that Eq. (4) should be modified to

$$\frac{P_1}{P_2} = \left(\frac{Q_1}{Q_2}\right)^{\alpha}$$
(5)

where α is a variable that depends on relevant parameters pertaining to the pump and the system. Equation (5) suggests the possibility of constructing a power consumption chart of a variable speed pump. This chart should give a better estimate of energy saving by variable speed control than the affinity laws alone. The main objective of this paper is to construct such a chart by using simplified models of pump performance curves and system curve.

2. Model Pump Characteristics

Pump performance curves at different speeds are usually supplied by manufacturers. When these curves are available, it is quite straightforward to obtain pump performance equations. In many instances, however, only data available for a pump are the design head (H_d) , the design flow rate (Q_d) , the maximum efficiency (η_d) at the design operation point, the maximum head (H_m) , and the corresponding flow rate (Q_m) . Obviously, model pump characteristics are needed in order to obtain pump performance equations in such a situation. Ulanicki et al. [9] suggest that pump head and efficiency may be taken to be polynomial functions of flow rate as follows:

$$H = aQ^2 + bQ + c \tag{6}$$

$$\eta = dQ^3 + eQ^2 + fQ + g \tag{7}$$

Values of unknown parameters in Eq. (6) are determined from (1) known values of H_d and Q_d , (2) known values of H_m and Q_m , and (3) the zero slope of H function at $Q = Q_m$. Values of unknown parameters in Eq. (7) are determined from (1) known values of η_d and Q_d , (2) the zero slope of η function at $Q = Q_d$, (3) zero value of η at Q = 0, and (4) zero value of η at $Q = Q_m$.

In order to reduce the number of parameters affecting the performance curves, it is useful to convert Eqs. (6) and (7) to dimensionless equations. Let's define dimensionless flow rate and head as

$$q = \frac{Q}{Q_0}$$
 (8)

$$h = \frac{H}{H_m}$$
(9)

where Q_0 is the maximum flow rate at which head is zero. Figure 3 shows curves of model pump head and efficiency as functions of flow rate. The dimensionless equations of pump head and efficiency are

$$h = \frac{(1-q)(1+q-2q_m)}{(1-q_m)^2}$$
(10)

$$\eta = \frac{q(1-q)(2q_dq - q + 2q_d - 3q_d^2)\eta_d}{q_d^2(1-q_d)^2}$$
(11)

Equations (8) and (9) require the value of Q_0 , which may be computed from Eq. (10). The expression for Q_0 is

$$Q_0 = (Q_d - Q_m) \sqrt{\frac{H_m}{H_m - H_d}} + Q_m \quad (12)$$



Fig. 3 Model pump head curve and pump efficiency curve.

Once Q, H, and η are known, power required is computed from

$$P = \frac{\gamma QH}{\eta}$$
(13)

where γ is specific gravity. Dividing *P* by $\gamma Q_0 H_m$ results in dimensionless power:

$$p = \frac{qh}{\eta}$$
(14)

Insert Eqs. (10) and (11) into Eq. (14). The resulting expression for dimensionless power is

$$p = \frac{(1+q-2q_m)q_d^2(1-q_d)^2}{(2q_dq-q+2q_d-3q_d^2)(1-q_m)^2\eta_d}$$
(15)

It is interesting to note that p is independent of q when

$$q_d = \frac{2q_m + \sqrt{4q_m^2 - 6q_m + 3}}{3}$$
(16)

In fact, *p* is a monotonically increasing function of *q*. Therefore, a physically meaningful value of q_d must be less than the value given in Eq. (16). For example, if $q_m = 0.2$, q_d must be less than 0.6. Figure 4 compares dimensionless power curves for different values of q_d .



Fig. 4 Dimensionless pump power curves for $q_m = 0.2$ and different values of q_d . Since power must increase monotonically with flow rate, q_d must be less than 0.6.

Let Eqs. (10) and (11) be pump characteristics at the speed of N_1 . Pump characteristics at the speed of N_2 are given by [9]

$$\frac{h}{n^2} = \frac{(1-q')(1+q'-2q_m)}{(1-q_m)^2}$$
(17)

$$\eta = \frac{q'(1-q')(2q_dq'-q'+2q_d-3q_d^2)\eta_d}{q_d^2(1-q_d)^2}$$
(18)

$$\frac{p}{n^3} = \frac{(1+q'-2q_m)q_d^2(1-q_d)^2}{(2q_dq'-q'+2q_d-3q_d^2)(1-q_m)^2\eta_d}$$
(19)

where $n = N_2/N_1$ and q' = q/n. Pump characteristics at two different speeds $(N_1 \text{ and } N_2)$ are shown in Fig. 5.



Fig. 5 Pump characteristics at pump speeds N_1 (solid lines) and N_2 (dashed lines).

3. Power Consumption Chart

The system equation is assumed to be

$$h = \left(\frac{h_d - h_s}{q_d^2}\right)q^2 + h_s$$
 (20)

so that the intersection between the system curve and the pump head curve [Eq. (10)] is at the design flow rate q_d where power is $p_d = q_d h_d / \eta_d$. If a new flow rate q_n is desired, pump speed must be changed to *n*, which is the solution of

$$\frac{n^2 (1-q')(1+q'-2q_m)}{(1-q_m)^2} = \left(\frac{h_d - h_s}{q_d^2}\right) q^2 + h_s$$
(21)

Once *n* is known, the new pump power p_n can be determined from Eq. (19). Equation (5) is now rewritten as

$$\frac{P_n}{P_d} = \left(\frac{q_n}{q_d}\right)^{\alpha}$$
(22)

which gives the expression for α :

(

$$\alpha = \frac{\log(p_n/p_d)}{\log(q_n/q_d)}$$
(23)



Fig. 6 Power consumption chart of a variable speed pump for $q_m = 0$ and $h_d = 0.5$.

Parameters that influence α include q_n , q_d , h_s , q_m , and h_m . Figure 6 shows a plot of α as a function of q_n/q_d and h_s for $q_d = 0.5$, and $q_m = 0$. It can be seen that α decreases as q_n/q_d decreases, and that α decreases as h_s increases. Different plots for other values of for q_d and q_m can be easily drawn. It should be noted that q_m is usually close to zero, and q_d is limited within a small range. Other plots of α should therefore look similar to Fig. 6. The plot in Fig. 6 may be called the power consumption chart for variable-speed centrifugal pump. It should be used in place of Eq. (4) to estimate energy saving resulting from variable-speed control of centrifugal pump.

As an example of the use of Fig. 6, consider a water pump having $H_m = 32 \text{ m}, Q_m = 0 \text{ m}^3/\text{s}, H_d = 24$ m, $Q_d = 8.8 \times 10^{-3} \text{ m}^3/\text{s}$, and $\eta_d = 80\%$, operating in a system having $H_{e} = 6.4$ m. It is required to compute the pumping power when the flow rate is reduced by half. The pumping power at the design operating point is 2.59 kW. Calculation of Q_0 from Eq. (12) gives $Q_0 = 0.0176$ m³/s. Dimensionless parameters of the pump and system become $q_d =$ $0.5, h_d = 0.75, q_m = 0$, and $h_s = 0.2$. The value of α at $q_{y}/q_{d} = 0.5$ and $h_{s} = 0.2$ is 2.1. Therefore, the pumping power at the reduced flow rate is $2590(0.5)^{2.1} =$ 604 W. This result may be compared with the pumping power calculated by using the affinity laws, which is equal to $2590(0.5)^3 = 324$ W. It can be seen that the pump affinity laws lead to substantial underestimation of the pumping power.

Although results in this paper are intended for centrifugal pumps, extension to axial-flow pumps may be possible. Figure 7 illustrates the typical pump head curve of axial pumps. The shut-off head is at point A. There is also a local minimum at point B, and a local maximum at point C. The operation region of the axial pump lies on the right of point C where the slope is negative. This region can certainly modeled by a quadratic function. Although such a model is unrealistic on the left of point C as shown by the dashed line, negligible error is expected since the operation point is unlikely to lie in this region.

4. Conclusion

A model of centrifugal pump characteristics is proposed on the assumption that pump head is a quadratic function of flow rate, and pump efficiency is a cubic function of flow rate. Parameters required for the determination of the coefficients of the head and efficiency functions are design flow rate, design head, design efficiency, maximum head, and the corresponding flow rate. Pump power function is determined from the head and efficiency functions. It is found that the ratio of the design flow to the maximum flow rate is restricted within a range less the power function is unrealistic. Affinity laws are used to construct pump characteristics at an arbitrary pump speed. Model pump characteristics and system curve, which is assumed to be a quadratic function, are used to construct pump power consumption chart, which can be used to compute the power required to run a pump at an arbitrary speed.



Fig. 7 Head curve of axial-flow pump. Dashed line indicates pump head curve modeled by a quadratic function.

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