อัลกอริทึมการตรวจหาและแก้ไขความขรุขระเชิงความร้อนสำหรับช่องสัญญาณการบันทึกเชิงแม่เหล็กแบบแนวตั้ง

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บทคัดย่อ

ความขรุขระเชิงความร้อน (TA: thermal asperity) เป็นปัญหามากสำหรับกระบวนการตรวจหาข้อมูลในช่องสัญญาณการบันทึกเชิงแม่เหล็กแบบแนวตั้งโดยมีสาเหตุมาจากการชนกันระหว่างความขรุขระบนสื่อบันทึกกับหัวอ่านแบบ MR ซึ่งทำให้สัญญาณอ่านกลับที่ได้มีความผิดเพี้ยนเกิดขึ้น และส่งผลให้เกิดข้อผิดพลาดจำนวนมากภายในข้อมูลหนึ่งเซ็กเตอร์ (sector) บทความนี้ได้นำเสนออัลกอริทึมการตรวจหาและแก้ไข TA ซึ่งประกอบด้วยสองช่องสัญญาณที่ทำงานขนานกันคือ ช่องสัญญาณที่มีทาร์เก็ตแบบ $H_1(D)$ และช่องสัญญาณที่มีทาร์เก็ตแบบ $H_2(D)$ ซึ่งทำงานร่วมกับวงจรกรองแถบผ่านแบบ $(1 - D^2)$ โดยที่ทาร์เก็ต $H_2(D)$ นี้จะถูกออกแบบโดยตรงในขณะที่มีระบบมีผลกระทบของ TA โดยทั่วไปวงจรตรวจหาวีเทอร์บิ (VD: Viterbi detector) ที่ใช้กับช่องสัญญาณ $H_1(D)$ จะให้อัตราข้อผิดพลาดของบิต (BER: bit-error rate) ต่ำ เมื่อระบบไม่มีผลกระทบของ TA ในขณะที่วงจรตรวจหาวีเทอร์บิที่ใช้กับช่องสัญญาณ $H_2(D)$ จะให้ค่า BER ต่ำเมื่อระบบมีผลกระทบของ TA ดังนั้นในการถอดรหัสข้อมูล ระบบที่นำเสนอจะทำการเลือกบิตข้อมูลจากวงจรตรวจหาวีเทอร์บิทั้งสอง โดยจะขึ้นอยู่กับว่ามีการตรวจพบสัญญาณ TA หรือไม่ ผลการทดลองแสดงให้เห็นว่าอัลกอริทึมที่นำเสนอมีประสิทธิภาพดีกว่าอัลกอริทึมแบบที่ใช้กันทั่วไป และยังมีความทนทานต่อการเปลี่ยนแปลงของแอมพลิจูดสูงสุดของสัญญาณ TA

คำสำคัญ : วงจรกรองแถบผ่าน / การบันทึกเชิงแม่เหล็กแบบแนวตั้ง / การออกแบบทาร์เก็ตและอีควอไลเซอร์ / ความขรุขระเชิงความร้อน

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A Thermal Asperity Detection and Correction Algorithm for Perpendicular Magnetic Recording Channels

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Abstract

The thermal asperity (TA) defect resulting from the collision between an asperity and the magneto-resistive (MR) read head can distort the readback signal to the extent of causing possible sector read failure. This paper presents a new TA detection and correction algorithm for perpendicular magnetic recording channels. The proposed algorithm consists of two channels running in parallel, one for the $H_1(D)$ target, and the other for the $H_2(D)$ target equipped with a bandpass filter $1 - D^2$, where the $H_2(D)$ target is directly designed in the presence of a TA. The Viterbi detector (VD) in the $H_1(D)$ channel has a lower bit-error rate (BER) in the absence of a TA, whereas that in the $H_2(D)$ channel has a lower BER in the presence of a TA. Thus, the overall decoded bit stream is selected from these two VDs, depending on whether a TA is detected. Results indicate that the proposed algorithm yields lower BER than the existing one, and is robust to large peak TA amplitudes.

Keywords : Bandpass Filter / Perpendicular Magnetic Recording / Target and Equalizer Design / Thermal Asperity

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1. Introduction

To achieve very high storage capacity in hard disk drives, the magneto-resistive (MR) read heads have been employed in place of the inductive heads. In practice, the MR read head senses the change in a flux via the transitions of the magnetization pattern written on the disk surface, resulting in an induced voltage pulse called a transition pulse. When an asperity (or a surface roughness) comes into contact with the slider, both the surface of the slider and the tip of the asperity are heated, which results in an extra voltage transient known as thermal asperity (TA). The vulnerability of MR sensors to TA was identified shortly after their discovery [1].

Typically, a TA signal has a short rise time (50 - 160 ns) with a long decay time (1 - 5μs), and its peak TA amplitude is 2 - 3 times the peak of the readback signal [2-3]. Practically, the TA effect can cause a burst of errors, which could easily exceed the correction capability of the error-control code (ECC), and thus results in a sector read failure. As the recording density keeps increasing and as the flying height keeps decreasing, the TA effect becomes even more serious in future disk drives. Perpendicular recording is the current technology used in today's hard drives [4]. Consequently, a method to suppress the TA effect for perpendi cular recording channel is crucial.

Several TA detection and correction algorithms have been proposed in the literature to alleviate the TA effect. In general, the TA causes a shift in the baseline of the readback signal. The average value of the normal readback signal is zero, while that of the TA-affected readback signal is not. Therefore, Klaassen and van Peppen [5] proposed the TA detection by looking at the baseline of the averaged readback signal, whereas the TA correction was performed by use of a high-pass filter. Dorfman and Wolf [3], [6] proposed a method to combat with the TA effect by passing the TA-affected readback signal through a filter (1 - D), where D is a delay operator. This method has been tested with an EPR4 target in longitudinal recording channels, where the number of bits corrupted by the TA effect is significantly reduced. Nevertheless, this method is not suitable for a perpendicular recording channel because this channel has a d.c. component [4].

For perpendicular recording channels, Fatih and Erozan [7] proposed a TA detection and correction method by use of different low-pass and high-pass filters, whereas Mathew and Tjhia [8] proposed a simple threshold-based approach to detect and suppress the TA effect. Eventually, Kovintavewat and Koonkarnkhai [9] proposed a TA suppression method based on a least-squares fitting technique for perpendicular recording channels.

This paper proposes a new TA detection and correction algorithm for perpendicular recording channels, which consists of two channels running in parallel. One channel is matched to the target response $H_1(D)$, while the other is matched to the target response $H_2(D)$ equipped with a bandpass filter of the form (1 - D^2) [10] to suppress a TA. Furthermore, the $H_2(D)$ target and its corresponding equalizer are directly designed in the presence of a TA based on the minimum mean-squared error (MMSE) [11] approach. In practice, the Viterbi detector (VD) [12] in the $H_1(D)$ channel has a lower bit-error rate (BER) in the absence of a TA, whereas that in the $H_2(D)$ channel has a lower BER in the presence of a TA. Therefore, the overall decoded bit stream is selected from these two VDs, depending on whether a TA is detected.

This paper is organized as follows. After describing a channel model in Section 2, Section 3 explains a widely used TA model. Section 4 briefly describes

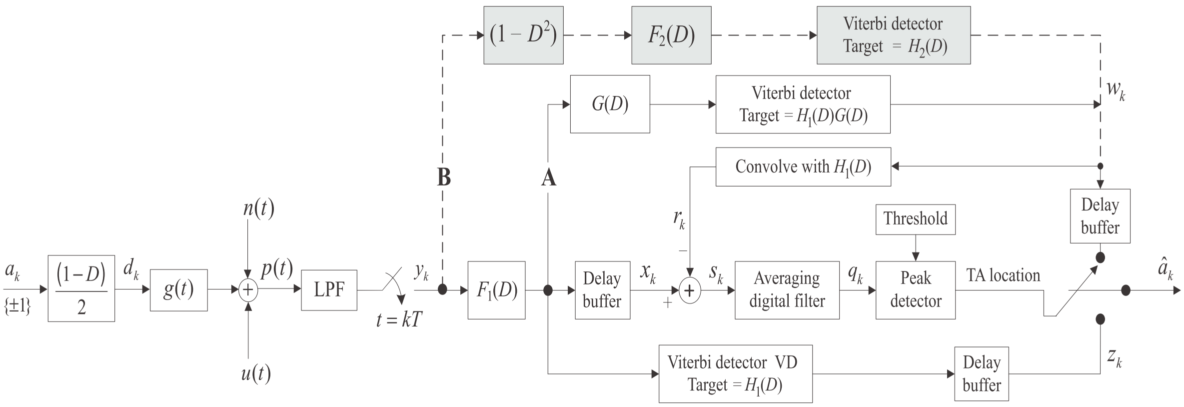

Fig. 1 A channel model with the different TA suppression methods, where the dashed line represents the proposed TA suppression method.

the target and equalizer design, and Section 5 presents the proposed TA suppression method. Simulation results are given in Section 6. Finally, Section 7 concludes this paper.

2. Channel Model

We consider the perpendicular recording channel illustrated in Fig. 1. A binary input sequence $a_k = \{\pm 1\}$ with bit period T is filtered by an ideal differentiator $(1 - D)/2$ to form a transition sequence $d_k \in \{-1, 0, 1\}$, where $d_k = \pm 1$ corresponds to a positive or a negative transition, and $d_k = 0$ corresponds to the absence of a transition. The transition sequence d_k passes through the magnetic recording channel represented by *g(t)*. The transition response *g(t)* for perpendicular recording is given by [13]

$$g(t) = \text{erf}\left(\frac{2t\sqrt{\ln 2}}{PW_{50}}\right), \quad \textbf{(1)}$$

where $\text{erf}(x) = (2/\sqrt{\pi})\int_0^x e^{-z^2}\,dz$ is an error function, and PW_{50} determines the width of the derivative of $g(t)$ at half its maximum. In the context of magnetic recording, a normalized recording density is defined as $\text{ND} = PW_{50}/T$, which determines how many data bits can be packed within the resolution unit PW_{50}.

The TA-affected readback signal, *p(t)*, can then be expressed as

$$p(t) = \sum_k d_k g(t - kT) + n(t) + u(t), \quad \textbf{(2)}$$

where $n(t)$ is additive white Gaussian noise (AWGN) with two-sided power spectral density $N_0/2$, and $u(t)$ is a TA signal. The signal $p(t)$ is filtered by a seventh-order Butterworth low-pass filter (LPF) and is then sampled at time $t = kT$, assuming perfect synchronization. The sampler output y_k is equalized by an equalizer, followed by the TA detection and correction block and the VD to determine the most likely input sequence.

3. Classical TA Model

This section briefly describes how to generate the TA signal, *u(t)*. Among many TA models proposed in the literature, we consider a widely used TA model described by Stupp *et al.* [2] as depicted in Fig. 2 because it fits captured spin stand data and drive data very well [7]. Typically, this classical TA signal has a short rise time with a long decay time, and its effect is assumed to decay exponentially. In general, this TA model is specified by four parameters as follows:

- START-TIME: It sets where the TA effect starts.
- RISE-TIME: It specifies the time required for the TA signal to rise from 0 to its maximum amplitude (defined by MAX-AMPLITUDE).
- MAX-AMPLITUDE: It determines the maximum amplitude of the TA signal.
- DECAY-TIME: It sets the time required for the TA signal to decay exponentially from its maximum amplitude to some small values.

Based on this TA model, we can model several TA scenarios that typify the conditions observed in product testing.

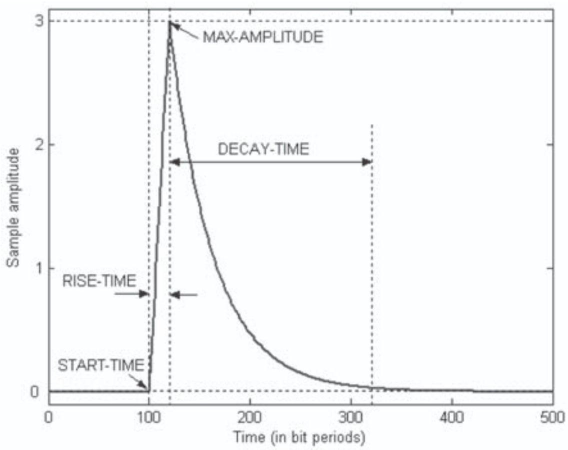

Fig. 2 A widely used TA signal, *u(t)*.

For instance, Fig. 3 illustrates the readback signal with different TA effects at the input of an LPF, where START-TIME = 400*T* and RISE-TIME = 10*T*, and DECAY-TIME = 200*T*. Clearly, immediately after the slider comes into contact with an asperity, the transient TA effect quickly and significantly changes the baseline of the read-back signal. Then, the slider and the asperity cool down so that the baseline of the signal decays to its original level. As displayed in Fig. 3, one would expect that the larger the values of MAX-AMPLITUDE and DECAY-TIME, the worse the system performance, as will be seen later in this paper.

In terms of mathematical expression, the TA signal in Fig. 2 can be expressed as [8]

$$u(t) = \begin{cases} A_0(t/T_r), & 0 \le t < T_r \\ A_0 \exp(-(t-T_r)/T_d), & T_r < t \le T_f \end{cases}, \quad (3)$$

where A_0 is the peak TA amplitude (or MAX-AMPLITUDE), T_r is a rise time (or RISE-TIME), and T_d is a decay constant. In this paper, the TA duration is assumed to be $T_f = T_r + 4T_d$ [8], where a decay time of $4T_d$ is sufficient because it will reduce the amplitude of the TA signal to approximately 1.8% of its peak amplitude.

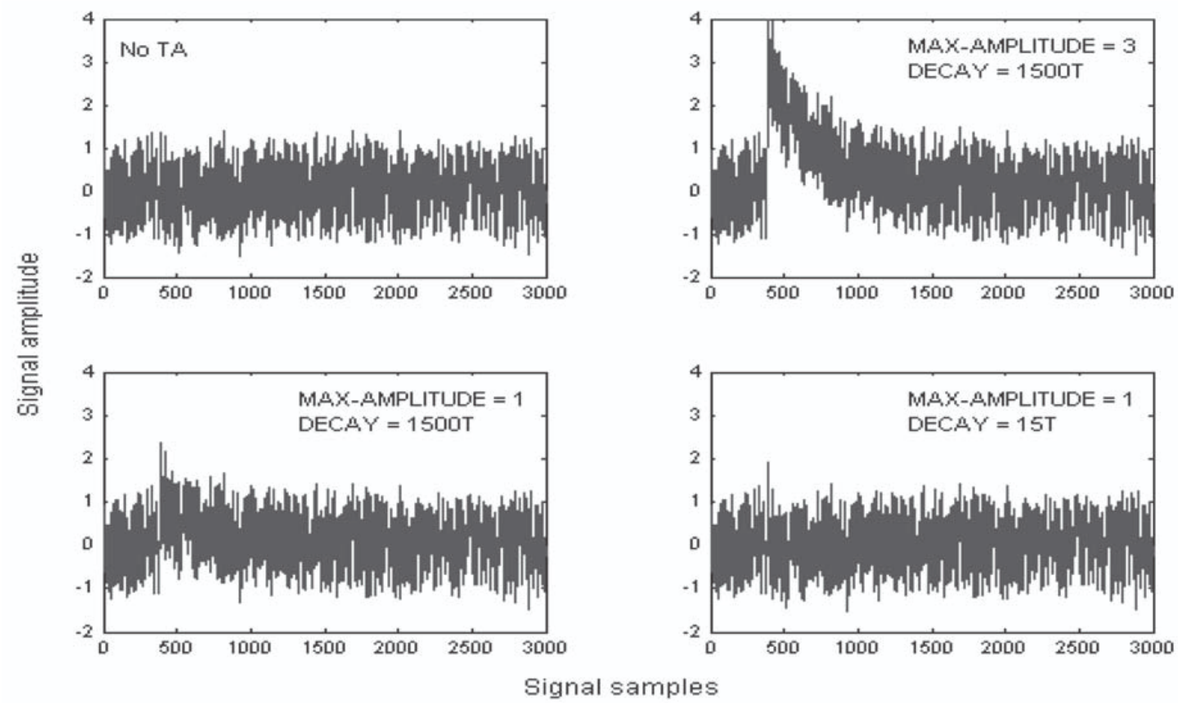

Fig. 3 Examples of the readback signal with different TA effects at the input of the low-pass filter.

4. Target and Equalizer Design

The target $H_1(D)$ and its corresponding equalizer $F_1(D)$ are simultaneously designed based on the MMSE approach, assuming that there is no TA in the system. Note that the resulting target obtained from this MMSE approach is known as the generalized partial response (GPR) target [11]. Thus, the two filters, $H_1(D)$ and $F_1(D)$, will be used to output the decoded bits $\{z_k\}$ when a TA is absent.

On the other hand, the target $H_2(D)$ and the equalizer $F_2(D)$ are simultaneously designed in the presence of a TA, based also on the MMSE approach according to Fig. 4, which can be obtained by minimizing

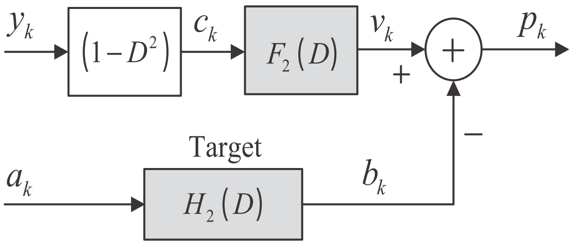

Fig. 4 Target and equalizer design for the proposed TA suppression algorithm.

$$E\{p_k^2\} = E\{[(c_k * f_k) - (a_k * h_k)]^2\}$$
$$= E\{[(y_k * f_k) - (y_{k-2} * f_k) - (a_k * h_k)]^2\} \quad (4)$$

where $E\{.\}$ is an expectation operator, $c_k = y_k - y_{k-2}$ is the input sequence of the $F_2(D)$ equalizer, h_k and f_k denote the filter coefficients of $H_2(D)$ and $F_2(D)$, respectively.

Let $\mathbf{H} = [h_0, h_1,...,h_{L-1}]^T$ denote the $H_2(D)$ target and $\mathbf{F} = [f_{-K},...,f_0,..f_K]^T$ represent the $F_2(D)$ equalizer, where L is the target length, $N = 2K + 1$ is the number of equalizer coefficients, and $[.]^T$ is the transpose operation. In this paper, $K = 5$ is employed in the GPR design with an assumption that the center tap is at $k = 0$. During the minimization process, we use the monic constraint [11], i.e., $h_0 = 1$, to avoid reaching the trivial solutions of $\mathbf{H} = \mathbf{F} = \mathbf{0}$. Thus, (4) can be rewritten as

$$E\{p_k^2\} = \mathbf{YFF}^T - 2\mathbf{TFF}^T - 2\mathbf{MFH}^T + \mathbf{RFF}^T + 2\mathbf{UFH}^T + \mathbf{AHH}^T - 2\lambda(\mathbf{I}^T\mathbf{H} - 1) \quad (5)$$

where $\mathbf{Y}$ is an N-by-N autocorrelation matrix of a sequence y_k, $\mathbf{T}$ is an N-by-N cross-correlation matrix of sequences y_k and y_{k-2}, $\mathbf{M}$ is an N-by-L cross-correlation matrix of sequences y_k and a_k, $\mathbf{R}$ is an N-by-N autocorrelation matrix of a sequence y_{k-2}, $\mathbf{U}$ is an N-by-L cross-correlation matrix of sequences y_{k-2} and a_k, $\mathbf{A}$ is an L-by-L autocorrelation matrix of a sequence a_k, λ is the Lagrange multiplier, and $\mathbf{I}$ is an L-element column vector whose first element is one and the rest is zero.

By differentiating (5) with respect to λ, $\mathbf{H}$, and $\mathbf{F}$, and setting the results to be zero, one obtains

$$\lambda = \frac{1}{\mathbf{I}^T[(-\mathbf{M}+\mathbf{U})^T\mathbf{X}(\mathbf{M}-\mathbf{U})+\mathbf{A}]^{-1}\mathbf{I}} \quad (6)$$

$$\mathbf{H} = \lambda[(-\mathbf{M}+\mathbf{U})\mathbf{X}(\mathbf{M}-\mathbf{U})+\mathbf{A}]^{-1}\mathbf{I} \quad (7)$$

$$\mathbf{F} = \mathbf{X}(\mathbf{M}-\mathbf{U})\mathbf{H}, \quad (8)$$

where $\mathbf{X} = (\mathbf{Y}\text{-}2\mathbf{T}+\mathbf{R})^{-1}$. In addition, it can be shown that λ in (6) is in fact the MMSE value for minimization process of (5).

The advantage of directly designing the target $H_2(D)$ and its corresponding equalizer $F_2(D)$ when a TA is present is that a better target can be obtained. Specifically, the VD in the $H_2(D)$ channel should provide a lower BER than that in the $H_1(D)G(D)$ channel in the presence of a TA. This could finally improve the overall system performance as will be seen in Section 6.

5. Proposed Algorithm

The proposed TA detection and correction algorithm has a similar structure as the one proposed in [3], as shown in Fig. 1, except that the branch **A** is replaced by the branch **B**. Apparently, the proposed method employs two VDs running in parallel, one for the $H_1(D)$ target, and the other for the $H_2(D)$ target equipped with a bandpass filter. The bandpass filter of the form $(1 - D^2)$ is proposed to mitigate the TA effect while maintaining most energy of the readback signal, because perpendicular recording channels have significant low-frequency content. Therefore, the overall decoded bit stream is chosen from the outputs of these two VDs. If a TA is detected, a decoded bit w_k is selected; otherwise, a decoded bit z_k is selected.

To detect a TA, a decoded sequence $\{w_k\}$ is convolved with the $H_1(D)$ target so as to obtain a sequence $\{r_k\}$, which approximates the readback signal. Then, the sequence $\{r_k\}$ is used to subtract the received sequence $\{x_k\}$ to obtain a sequence $\{s_k\}$, consisting of the predicted noise and the TA signal (if present). To remove the noise in a sequence $\{s_k\}$, an averaging digital filter is employed, which yields a sequence $\{q_k\}$ according to

$$q_k = \frac{1}{2\beta+1}\sum_{i=k-\beta}^{k+\beta} s_i \,, \quad (9)$$

where β is an integer, and $2\beta + 1$ is the window length for computing q_k. Finally, the peak detector determines the presence of the TA in a sequence $\{s_k\}$ and its location. This TA location will be utilized to select the decoded bit from $\{w_k\}$ or $\{z_k\}$ according to

$$\hat{a}_k = \begin{cases} w_k\,, & q_k \geq m \\ z_k\,, & q_k < m \end{cases}, \quad (10)$$

where m is a threshold. It should be noted that a large threshold will lead to a better AWGN performance at the expense of the TA performance. Conversely, a small threshold will lead to many false alarms, resulting in the output bit being w_k in the absence of a TA.

Based on extensive simulation search, we found that $m = 0.15$ and $\beta = 50$ are suitable parameters for this perpendicular recording channel because they can yield a good performance in the presence and in the absence of TAs.

6. Simulation Result

Consider a perpendicular recording channel at ND = 2.5. The signal-to-noise ratio (SNR) is defined as

$$\text{SNR} = 10\log_{10}\left(\frac{E_i}{N_0}\right) \quad (11)$$

in decibel (dB), where E_i is the energy of the channel impulse response (i.e., the derivative of the transition response scaled by 2). In simulation, every data sector is corrupted by one TA signal, which is occurred at the 1000-th bit with $A_0 = 2$, $T_r = 60$ ns, and $T_d = 0.5$ μs (i.e., a TA event $T_f = 1030T$). This TA event can be considered as a worst case. We compute the BER of the system based on a minimum number of 500 4096-bit data sectors and 500 error bits, and call that number as "BER given TA".

Table 1 The GPR targets used in simulation for each TA detection and correction algorithm.

Method	**Target** (when a TA is absent)	**Effective target** (when a TA is present)
M1 with $G(D) = 1 - D$	$1 + 1.3D + D^2 + 0.42D^3 + 0.09D^4$	$H_1(D)G(D) = 1 + 0.34D - 0.33D^2 - 0.58D^3 - 0.33D^4 - 0.09D^5$
M1 with $G(D) = 1 - D^2$	$1 + 1.32D + 0.92D^2 + 0.31D^3$	$H_1(D)G(D) = 1 + 1.32D - 0.08D^2 - 1.01D^3 - 0.92D^4 - 0.31D^5$
Proposed method	$1 + 1.32D + 0.92D^2 + 0.31D^3$	$H_2(D) = 1 + 0.95D + 0.10D^2 - 0.69D^3 - 0.78D^4 - 0.37D^5$

In this paper, the proposed TA suppression method is compared with the one proposed in [3], which is referred to as "M1." Based on the MMSE approach, the target $H_1(D)$ and its equalizer $F_1(D)$ are designed in the absence of a TA, whereas the target $H_2(D)$ and its equalizer $F_2(D)$ are designed in the presence of a TA using (6) - (8). In addition, we set all effective targets employed in the VD when a TA is present to be 6 taps. Table 1 shows the GPR targets used in simulations for each TA detection and correction algorithm.

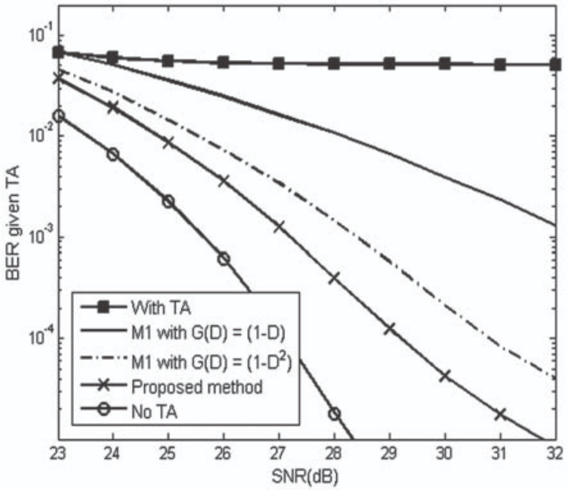

Fig. 5 BER performance at different SNRs.

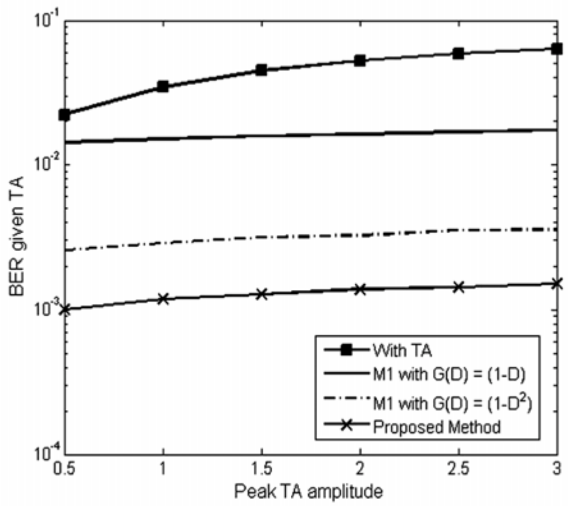

Fig. 6 BER performance with different peak TA amplitudes.

Fig. 5 compares the BER performance of different TA suppression methods as a function of SNR's, where the system performance in the absence of a TA is referred to as "No TA". It is evident that without the TA suppression method, the system performance is unacceptable (denoted as "With TA"), and the proposed method performs better than other methods.

We also compare the performance of different TA suppression methods as a function of peak TA amplitudes at SNR = 27 dB in Fig. 6, where the system without a TA event yields BER $\approx 10^{-4}$. It is apparent that the proposed method performs better than other methods, and is robust to large peak TA amplitudes.

7. Conclusion

The TA effect can distort the readback signal to the extent of causing a sector read failure. This paper proposes a new TA detection and correction algorithm to reduce the TA effect in perpendicular recording channels. The proposed method consists of two channels running in parallel, one for the $H_1(D)$ target, and the other for the $H_2(D)$ target equipped with a bandpass filter $(1 - D^2)$. Moreover, based on the MMSE approach, the target $H_1(D)$ was designed in the absence of a TA, while the target $H_2(D)$ was directly designed in the presence of a TA. It is evident from simulations that the proposed algorithm performs better than the method proposed in [3] for all peak TA amplitudes.

8. Acknowledgement

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