การประหยัดพลังงานของเครื่องสูบแบบหอยโข่งด้วยการลดขนาดใบพัด

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บทคัดย่อ

การลดขนาดใบพัดเป็นวิธีลดอัตราการไหลและกำลังงานของเครื่องสูบแบบหอยโข่งที่ง่ายและมีประสิทธิผล กฎความคล้ายที่รู้จักกันเป็นอย่างดีระบุว่า กำลังงานของเครื่องสูบแปรผันตามขนาดของใบพัดยกกำลังสามและอัตราการไหลแปรผันตามขนาดของใบพัด ดังนั้นการใช้กฎความคล้ายโดยตรงทำให้ได้ข้อสรุปที่ว่า กำลังงานจะแปรผันตามอัตราการไหลยกกำลังสาม อย่างไรก็ตามข้อสรุปนี้อาจจะนำไปสู่การประเมินการประหยัดพลังงานจากการลดขนาดใบพัดที่ผิดพลาดได้ บทความนี้นำเสนอวิธีที่แม่นยำขึ้นในการประมาณค่าพลังงานที่ประหยัดได้ วิธีนี้ใช้เส้นโค้งสมรรถนะที่ได้จากผู้ผลิตเครื่องสูบในการประมาณกำลังงานของเครื่องสูบที่มีใบพัดขนาดต่างๆ เส้นโค้งของเฮดและประสิทธิภาพของเครื่องสูบอาจประมาณด้วยฟังก์ชันกำลังสอง นอกจากนี้เส้นโค้งระบบก็เป็นฟังก์ชันกำลังสองเช่นกัน วิธีนี้สามารถสร้างเส้นโค้งกำลังงานของเครื่องสูบที่ค่าเฮดสถิตต่างๆ ได้ การวิเคราะห์พลังงานที่ประหยัดได้จากการลดขนาดใบพัดแสดงให้เห็นว่า การใช้กฎความคล้ายโดยตรงในการประมาณการประหยัดพลังงานอาจใช้ได้ในกรณีที่เฮดสถิตมีค่าเท่ากับศูนย์หรือมีค่าน้อยมาก แต่จะทำให้ประเมินพลังงานที่ประหยัดได้มากเกินความเป็นจริงเกินไปในกรณีที่เฮดสถิตมีค่ามาก

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Energy Saving in Centrifugal Pumps by Impeller Trimming

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Abstract

Impeller trimming is a simple and effective method to reduce the flow rate and the power input of a centrifugal pump. The well-known affinity laws for centrifugal pumps state that power input varies with the cube of impeller diameter, and flow rate varies with impeller diameter. This means that the direct application of the affinity laws leads to the conclusion that power input varies with the cube of flow rate. However, this conclusion may lead to an inaccurate estimation of energy saving from impeller trimming in practice. This paper presents a method for providing a better estimate of energy saving. This method uses pump performance curves supplied by pump manufacturers to estimate the power input required by a pump having an arbitrary impeller size. It is shown that both head curve and efficiency curve can be approximated by quadratic functions. Together with the assumption that the system curve is also a quadratic function, the presented method is capable of producing curves of pump power input at selected values of static head. The analysis of energy saving from impeller trimming shows that the direct application of the affinity laws to estimate energy saving may be used when the static head is zero or very small. A large static head, however, may give rise to a significant overestimation of energy saving.

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1. Introduction

Centrifugal pumps increase the pressure of fluid using rotational power input from a motor or an engine. The fluid flows axially into the pump impeller, which consists of many blades, and centrifugal forces of the rotating blades force the radial outflow of the fluid. Centrifugal pumps come in a wide variety of sizes, designs, and capacities. The performance of each pump is characterized by the increase in fluid energy or pump head, the power required to operate the pump, and the pump efficiency. These three parameters vary with the fluid flow rate. Testing of the pump will reveal how pump head, power, and efficiency vary with flow rate. Test results are usually used to draw pump performance curves.

It is well known that the performance of a centrifugal pump is affected by the diameter of the pump impeller. For the purpose of simulation and optimization, not only is a mathematical model of the pump performance curve needed, but a model of how the pump performance curve changes as the diameter of the impeller changes is also required. Obviously, such a model can be obtained by carrying out pump testing at different impeller diameters. However, such a test is time-consuming, expensive, and inconvenient. Therefore, it is usually assumed that the pump affinity laws can be applied, which results in construction of another performance curve at an arbitrary impeller diameter from an original performance curve. However, the effect of impeller diameter on the pump performance curves according to the affinity laws is unclear. Dimensional analysis reveals that different effects result from different assumptions on the similarity between two pumps having different impeller diameters.

Centrifugal pumps are common machinery found in virtually all factories and large buildings. Reduction of energy consumption by centrifugal pumps will, therefore, make a substantial contribution toward any energy conservation effort in a factory or building. It is a standard practice to specify an oversized pump during the design stage to allow for either future expansion or unforeseen losses. An oversized pump may deliver too much flow rate, and necessitate the use of a throttle valve to reduce flow rate. Such a practice results in energy inefficiency. A more energy efficient way to reduce flow rate is by using variable-speed control [1 - 3]. However, this method requires a large investment. An alternative and cheaper method is reducing the impeller diameter or impeller trimming [4, 5]. There has been suggestion that pump power varies with the cube of impeller diameter, and flow rate varies with impeller diameter [6]. This implies that pump power varies with the cube of flow rate. According to this suggestion, substantial energy saving is to be expected from impeller trimming. However, the validity of this suggestion is questionable due to the uncertainty of the effect of impeller diameter on the pump performance curves. The main objective of this paper is to investigate how pump power input varies with impeller diameter in five commercial centrifugal pumps using actual pump performance data supplied by the pump manufacturers.

2. Affinity Laws

Head (H) and power (P) of a centrifugal pump are functions of several parameters. If effects of viscosity and fluid compressibility are assumed to be negligible, the remaining parameters are flow rate (Q), pump speed (N), pump size (D), fluid density (ρ), and gravitational acceleration (g). Dimensional analysis results in the construction of

3 dimensionless parameters: flow coefficient (q), head coefficient (h), and power coefficient (p). Expressions of these dimensionless parameters are [7]

$$q = \frac{Q}{ND^3} \quad (1)$$

$$h = \frac{gH}{N^2 D^2} \quad (2)$$

$$p = \frac{P}{N^3 D^5} \quad (3)$$

Functional relationships among these parameters may be written as

$$h = f_1(q) \quad (4)$$

$$p = f_2(q) \quad (5)$$

Consider 2 geometrically similar pumps with 2 different sizes (D_1 and D_2) operating at the same pump speed. Equations (4) and (5) imply that

$$\frac{Q_1}{Q_2} = \left(\frac{D_1}{D_2}\right)^3 \quad (6)$$

$$\frac{H_1}{H_2} = \left(\frac{D_1}{D_2}\right)^2 \quad (7)$$

$$\frac{P_1}{P_2} = \left(\frac{D_1}{D_2}\right)^5 \quad (8)$$

It should be noted that Eqs. (6) - (8) are not applicable for the same pump operating with 2 impeller diameters. In this case, there is more than one characteristic length to consider, and the dimensional analysis leading to Eqs. (6) - (8) may have to be revised. Fig. 1 illustrates a model pump impeller. It can be seen that there are 3 characteristic lengths: inner diameter (d), outer diameter (D), and impeller width (b). Two new lengths (d and b) are of relevance because fluid flows into the impeller at the inner diameter (where the flow area is πdb) and flows out of the impeller at the outer diameter (where the flow area is πDb). It is well known centrifugal forces exerted by blades (not shown) in the annular region of Fig. 1 on fluid cause an increase in pump head.

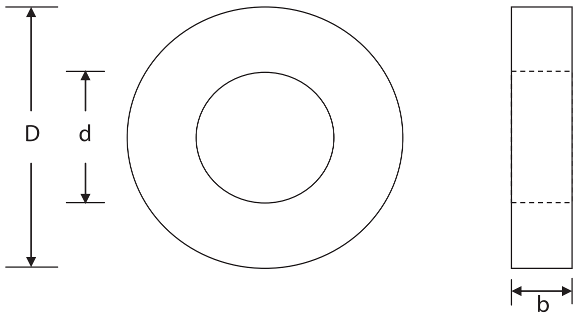

Fig. 1 Characteristic lengths of model pump impeller

With the inclusion of all three characteristic lengths, the dimensional analysis can be simplified by assuming that the area scale D^2 in Eq. (1) may be replaced by the flow area (A). As a result, the flow coefficient becomes

$$q = \frac{Q}{NDA} \quad (9)$$

The head coefficient needs no revision because the denominator of Eq. (2) is the characteristic blade velocity where the fluid leaves the impeller. Now the question is which area to use in Eq. (9). If the area is the exit area, A = πDb, Eq. (9) becomes

$$q = \frac{Q}{\pi bND^2} \quad (10)$$

If the area is the inlet area, A = πdb, Eq. (9) becomes

$$q = \frac{Q}{\pi bdND} \quad (11)$$

Note that impeller trimming reduces D, but not d. If b, d, and N are assumed to be constant, the relationship between H and Q in Eq. (4) may be rewritten as

$$\frac{H}{D^2} = f_3\left(\frac{Q}{D^k}\right) \quad (12)$$

where k equals 2 according to Eq. (10) or 1 according Eq. (11). Consequently, Eqs. (6) and (8) become [5]

$$\frac{Q_1}{Q_2} = \left(\frac{D_1}{D_2}\right)^2 \quad (13)$$

$$\frac{P_1}{P_2} = \left(\frac{D_1}{D_2}\right)^4 \quad (14)$$

when k = 2, or become [6]

$$\frac{Q_1}{Q_2} = \frac{D_1}{D_2} \quad (15)$$

$$\frac{P_1}{P_2} = \left(\frac{D_1}{D_2}\right)^3 \quad (16)$$

when k = 1.

3. Pump Performance Curves

In order to verify that the affinity laws are applicable to actual pumps, and to determine the value of k that should be used, it is imperative to consult experimental data. However, few experimental data on impeller trimming are available. Savar et al. [5] performed an experiment on impeller trimming, in which pump heads and flow rates of a centrifugal pump were measured for each of 7 impeller diameters, and used the results to draw a performance curve for each impeller diameter. Li [8] investigated the performance of a commercial oil pump with the original impeller trimmed 4 times. The results of his experiment show that the affinity law fails to predict the performance of pumps that handle viscous oil, and that the modification of the affinity law must take into account the viscosity of the liquid. Experimental data supplied by Savar et al [5] and Li [8] are not sufficient for this study. Therefore, it is necessary to obtain other sources of data. Pump manufacturers have tested their pumps extensively, and used them to construct pump performance curves. Pump performance curves consist of head curve, power curve, and efficiency curve showing variations of pump head, power, and efficiency, respectively, with flow rate. Most manufacturers supply only head curves, along with superimposed iso-efficiency lines, because power curves can be easily drawn from the provided curves. For a pump model, manufacturers usually have different impeller sizes for their customers to choose from, and provide head curves for the available impeller diameters. It may be assumed that an impeller of a reduced size is identical with a trimmed impeller of the same diameter. Five pump manufacturers that make their pump performance curves available for download are

- Aurora (http://www.aurorapump.com)
- Bell & Gossett (http://www.bellgossett.com)
- Goulds Pumps (http://www.gouldspumps.com)
- KSB (http://www.ksb.com)
- GEA Tuchenhagen (http://www.tuchenhagen.com)

Table 1 gives details of five centrifugal pump models from five manufacturers chosen for this study due to their on-line availability. It should be noted that more head curves of other impeller diameters may be available from the manufacturers, but only head curves of impeller diameters listed in Table 1 are chosen for this study because it has been suggested that trimming should be limited to about 75% of the maximum impeller diameters [9].

Table 1 Details of 5 pump models chosen for this study

Pump Number	Manufacturer	Model	Impeller Sizes (m)	Speed (rpm)
1	Aurora	410 Size 2x2 1/2x10	0.241, 0.216, 0.191	2880
2	Bell & Gossett	4x6x10M HSC	0.305, 0.287, 0.269, 0.251, 0.234	3565
3	Goulds Pumps	3656/3756 S Group	0.151, 0.143, 0.130, 0.121	2900
4	KSB	Elite E32-13	0.139, 0.130, 0.120	2900
5	GEA Tuchenhagen	TP 1020	0.130, 0.120, 0.110	2900

Fig. 2 shows generic pump performance curves that can be obtained from pump manufacturers listed in Table 1. Quick inspection of Fig. 2 reveals that a head curve may be approximated by a quadratic function, which is also suggested by Chantasiriwan [3], Benier and Lemire [10], and Ulanicki et al. [11]. Furthermore, if the affinity laws are applicable, head curves for different impeller diameters will collapse into a single curve [10]. Equation (12) implies that the functional relationship between head and flow rate may be written as

$$H\left(\frac{D_1}{D}\right)^2 = a_1Q^2\left(\frac{D_1}{D}\right)^{2k} + a_2Q\left(\frac{D_1}{D}\right)^k + a_3 \qquad \textbf{(17)}$$

where D_1 is the largest impeller diameter. D_1 is 0.241 m, 0.305 m, 0.151 m, 0.139 m and 0.130 m for Pumps #1, #2, #3, #4 and #5, respectively. Since it is uncertain what k should be, it is considered as a variable like a_1, a_2, and a_3. The functional form of the efficiency curve is less obvious. Chantasiriwan [3] and Ulanicki et al. [11] suggest that it should be approximated as a cubic function of flow rate because efficiency must be zero at zero flow rate and maximum flow rate in addition to being maximum at the optimum or design flow rate. However, the range of pump efficiency of interest is usually limited to the range near the maximum efficiency. It can be shown that, in this range, the quadratic approximation is quite satisfactory. Therefore, it can be assumed that

$$\eta = b_1Q^2\left(\frac{D_1}{D}\right)^{2k} + b_2Q\left(\frac{D_1}{D}\right)^k + b_3 \qquad \textbf{(18)}$$

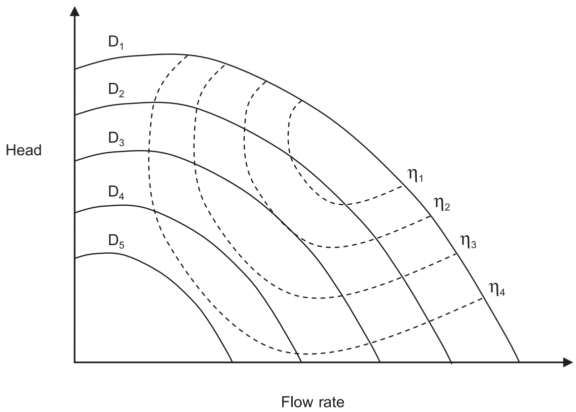

Fig. 2 Generic pump performance curves provided by pump manufacturers

Variables in Eq. (17) are determined from performance curves supplied by manufacturers listed in Table 1. Ten data points are extracted from a head curve for impeller diameter D. Each data point (Q, H) is then converted to $(Q(D_1/D)^k, H(D_1/D)^2)$, and placed on a new plot as shown in Fig. 3. Let the total number of extracted data points be n. Equation (17) may be written as

$$y = a_1x^2 + a_2x + a_3 \qquad \textbf{(19)}$$

where x represents $Q(D_1/D)^k$, and y represents $H(D_1/D)^2$. The sum of the squares of the residuals is

$$S_r = \sum_{i=1}^{n}(y_i - a_1x_i^2 - a_2x_i - a_3)^2 \qquad \textbf{(20)}$$

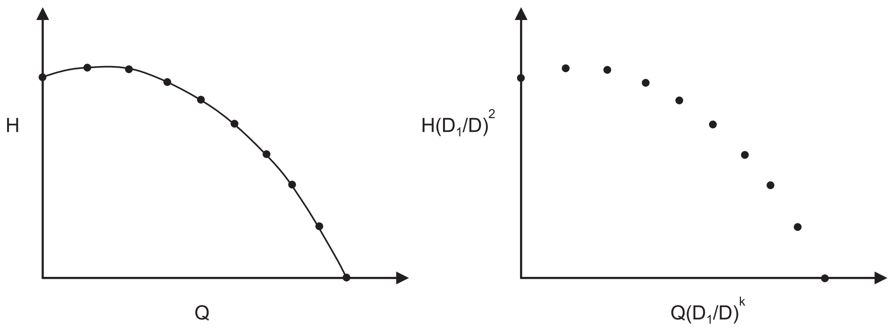

Fig. 3 Data extraction of head curve

In order to find optimum values of a_1, a_2 and a_3, S_r is minimized, leading to 3 linear algebraic equations:

$$a_1\sum_{i=1}^{n} x_i^2 + a_2\sum_{i=1}^{n} x_i + a_3 n = \sum_{i=1}^{n} y_i \quad \textbf{(21)}$$

$$a_1\sum_{i=1}^{n} x_i^3 + a_2\sum_{i=1}^{n} x_i^2 + a_3\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i y_i \quad \textbf{(22)}$$

$$a_1\sum_{i=1}^{n} x_i^4 + a_2\sum_{i=1}^{n} x_i^3 + a_3\sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i^2 y_i \quad \textbf{(23)}$$

which can be easily solved for a_1, a_2 and a_3. It is interesting to note that a_1, a_2, and a_3 depend on k. For each k (and corresponding a_1, a_2 and a_3), correlation coefficient is computed as follows.

$$r = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{i=1}^{n} (y_i - \bar{y})^2}} \quad \textbf{(24)}$$

where $\bar{x} = \Sigma x_i/n$ and $\bar{y} = \Sigma y_i/n$. Since a large value of r indicates a good fit, the optimum value of k is the one that yields maximum r. Although it is possible to search for the optimum value of k by performing repeated computations of r using several values of k, it is decided that there is no need for this. Only three values of k are considered: 1, 1.5, and 2. Fig. 4 shows that the parabolas fit to data points quite well. It is found that k equals 1.5 for pumps #1, #3 and #4; and k equals 1 for pump #2 and #5. Table 2 list values of a_1, a_2 and a_3 for the 5 pump models along with correlation coefficients, which are quite high in all cases.

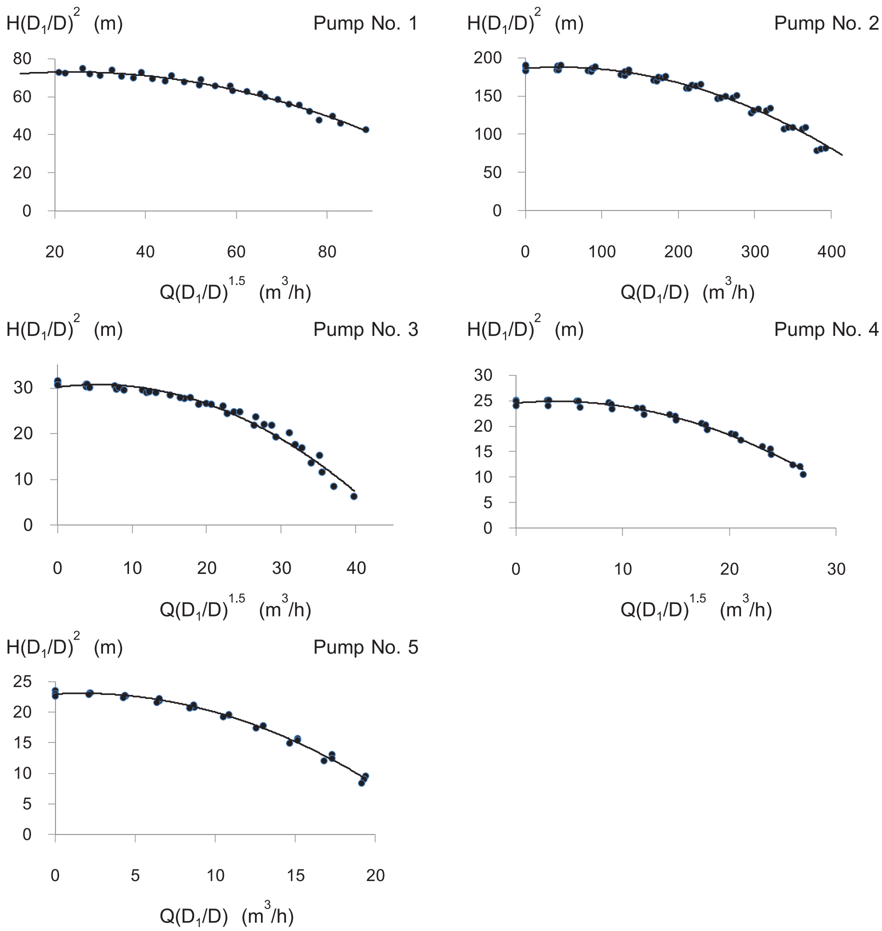

Fig. 4 Curve fitting of head curves of 5 pump models

Table 2 Values of coefficients in Eq. (13) from curve fitting

Pump Number	a_1	a_2	a_3	r
1	-0.0074	0.3498	69.35	0.989
2	-0.0008	0.0736	186.5	0.992
3	-0.0198	0.2158	30.24	0.988
4	-0.0246	0.1822	24.57	0.993
5	-0.0444	0.1482	23.01	0.995

Similar procedure is used to determine b_1, b_2, and b_3 in Eq. (18). Data points are extracted from an iso-efficiency for impeller diameter D. Each data point (Q, η) is then converted to $(Q(D_1/D)^k, \eta)$ with k fixed at the value obtained from the determination of a_1, a_2 and a_3, and placed on a new plot as shown in Fig. 5. Curve fitting is then performed for each pump model to determine b_1, b_2 and b_3. Results are shown in Fig. 6. It can be seen that the parabolas fit to data points quite well. Table 3 lists values of b_1, b_2 and b_3 for the 5 pump models along with correlation coefficients.

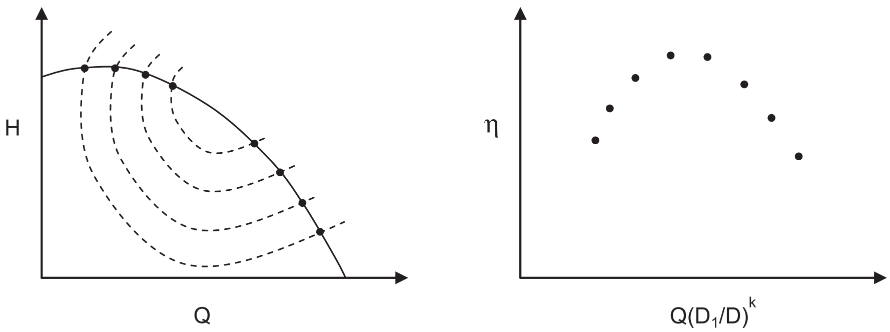

Fig. 5 Data extraction of efficiency curve

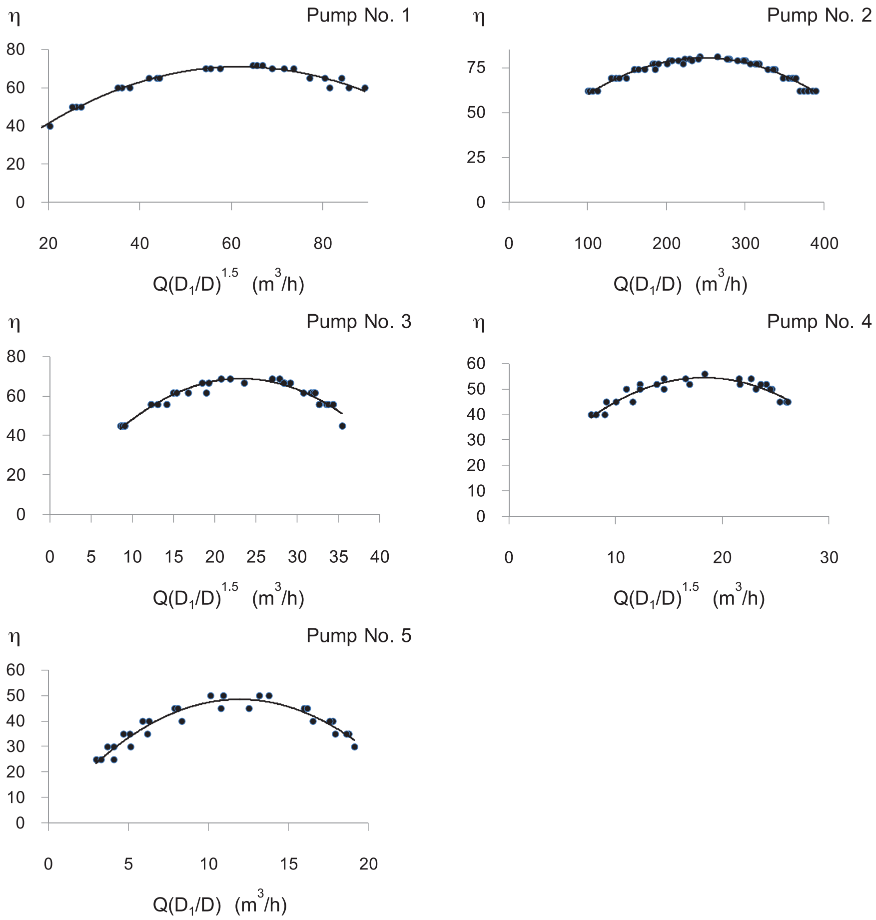

Fig. 6 Curve fitting of efficiency curves of 5 pump models

4. Model of Energy Saving by Impeller Trimming

It is assumed that, originally, a pump is designed to operate with the largest impeller (D_1) at the design point (Q_D, H_D), where efficiency is maximum. Since η in Eq. (18) can be considered as a function of $Q(D_1/D)^k$, the expression for Q_D can be found by maximizing η with respect to $Q(D_1/D)^k$. The resulting design flow rate is

$$Q_D = \frac{-b_2}{2b_1} \quad \textbf{(25)}$$

Correspondingly, H_D and η_D (the maximum efficiency) can be found by inserting Q_D into Eqs. (17) and (18), respectively, and setting $D = D_1$.

$$H_D = a_1Q_D^2 + a_2Q_D + a_3 \quad \textbf{(26)}$$

$$\eta_D = b_1Q_D^2 + b_2Q_D + b_3 \quad \textbf{(27)}$$

The power requirement at the design operation point is

$$P_D = \frac{\rho g Q_D H_D}{\eta_D} \quad \textbf{(28)}$$

Numerical values of Q_D, H_D, η_D and P_D for the 5 pump models are shown in Table 4.

Table 3 Values of coefficients in Eq. (14) from curve fitting

Pump Number	b_1	b_2	b_3	r
1	-0.0169	2.0838	6.636	0.990
2	-0.0009	0.4654	23.06	0.978
3	-0.1195	5.552	4.637	0.966
4	-0.1411	5.159	7.573	0.926
5	-0.3110	7.446	4.068	0.948

Table 4 Flow rate, head, efficiency, and power requirement for 5 pump models at design operation points.

Pump Number	Q_D (m^3/h)	H_D (m)	η_D (%)	P_D (kW)
1	61.65	62.79	70.87	14.88
2	258.6	152.0	83.23	128.7
3	23.23	24.57	69.12	2.250
4	18.28	19.68	54.73	1.791
5	11.97	18.42	48.64	1.236

The design flow rate is assumed to be too high, and it is desired to reduce flow rate from Q_D to Q_N by using a smaller impeller. This situation is illustrated in Fig. 7. In order to find the impeller size that gives the desired flow rate, it is necessary to make an assumption about the system curve. It has been suggested that the system curve should be approximated by a parabola. This suggestion is adopted in this paper, and the system equation is assumed to be

$$H = KQ^2 + H_S \quad \textbf{(29)}$$

where K depends on the amount of friction in the system, and H_S is the static head of the system. The static head is considered to be a free parameter, whereas the K can be determined from the fact that the design point (Q_D, H_D) lies on the system curve.

$$K = \frac{(H_D - H_S)}{Q_D^2} \quad \textbf{(30)}$$

Given the desired flow rate (Q_N), pump head at the new operation point (H_N) can be computed from Eqs. (29) and (30).

$$H_N = \frac{(H_D - H_S)}{Q_D^2}Q_N^2 + H_S \quad \textbf{(31)}$$

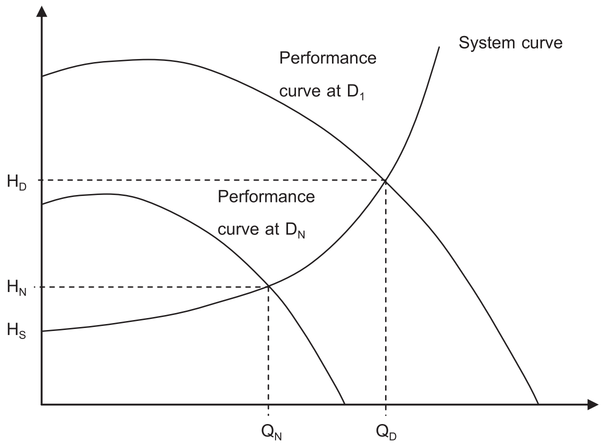

Fig. 7 Using impeller trimming to reduce the design flow rate (Q_D) to a new flow rate (Q_N)

Replace Q, H and D in Eq. (17) by Q_N, H_N and D_N, respectively.

$$H_N\left(\frac{D_1}{D_N}\right)^2 = a_1Q_N^2\left(\frac{D_1}{D_N}\right)^{2k}+a_2Q_N\left(\frac{D_1}{D_N}\right)^{k}+a_3 \quad \textbf{(32)}$$

Next, insert the expression for H_N from Eq. (31) in Eq. (32). The result is an equation that can be solved for DN.

$$\left[\frac{(H_D-H_S)}{Q_D^2}Q_N^2+H_S\right]\left(\frac{D_1}{D_N}\right)^2 = a_1Q_N^2\left(\frac{D_1}{D_N}\right)^{2k}+a_2Q_N\left(\frac{D_1}{D_N}\right)^{k}+a_3 \quad \textbf{(33)}$$

It should be noted that if k = 1, Eq. (33) becomes a quadratic equation, and the formula for finding its roots is well known. However, Eq. (33) is a nonlinear equation without a known formula for finding its roots if k = 1.5, and an iterative method must be used to find the solution. First a guess of the value of D_N/D_1 (which is less than 1) must be made. Then the updated value of D_N/D_1 is computed from the rearranged expression from Eq. (33).

$$\frac{D_N}{D_1} = \sqrt{\frac{\left[(H_D-H_S)(Q_N/Q_D)^2+H_S-a_1Q_N^2(D_N/D_1)^{2-2k}-a_2Q_N(D_N/D_1)^{2-k}\right]}{a_3}} \quad \textbf{(34)}$$

The iterative process is then repeated until there is a negligible change in the value of D_N/D_1. Once D_N has been found, pump power at the new operation point can be computed from

$$P_N = \frac{\rho g Q_N H_N}{\eta_N} \quad \textbf{(35)}$$

where

$$\eta_N = b_1Q_N^2\left(\frac{D_1}{D_N}\right)^{2k}+b_2Q_N\left(\frac{D_1}{D_N}\right)^{k}+b_3 \quad \textbf{(36)}$$

and H_N is computed from Eq. (31).

5. Results and Discussion

The procedure described in Section 4 can now be used to compute the power consumption and the impeller diameter required of a pump to deliver a given flow rate. For example, suppose that the flow rate of Pump #1 is to be reduced from the design flow rate to a new flow rate of 43.16 m^3/h, and that the static head of the system curve is 12.56 m. According to Table 2, a_1 = -0.0074, a_2 = 0.3498, and a_3 = 69.35. According to Table 4, QD = 61.65 m^3/h, and HD = 62.79 m. Furthermore, it is found earlier that k = 1.5. (See Fig. 4.) Therefore, all parameters in Eq. (34) are known, and the solution for D_N/D_1 is found to be 0.800. This means that the required impeller diameter is 0.193 m. According to Table 3, b_1 = -0.0169, b_2 = 2.0838, and b_3 = 6.636. P_N can then be computed from Eq. (35), and is found to be 6.696 kW.

It is quite inconvenient to go through the procedure described above. Therefore, Figs. 8 - 12 are constructed in order to show how P_N/P_D varies with Q_N/Q_D at selected static heads (HS) and impeller diameters (D) for Pump #1 - #5, respectively. Five selected static heads are 0, $0.1H_D$, $0.2H_D$, $0.3H_D$, $0.4H_D$, $0.5H_D$. Selected impeller diameters are $0.8D_1$, $0.85D_1$, $0.9D_1$, $0.95D_1$ for Pump #1 - #3 because ratios of smallest impeller sizes to largest impeller sizes in Table 1 are approximately 0.8. Selected impeller diameters are $0.85D_1$, $0.9D_1$, $0.95D_1$ for Pump #4 and #5 because ratios of smallest impeller sizes to largest impeller sizes in Table 1 are approximately 0.85. It can be seen that all figures display the same trend. Each curve at a fixed D/D_1 shows the monotonic increase in pump power with flow rate, which corresponds to a model presented by Chantasiriwan [3]. Each curve at a fixed H_S/H_D also shows the monotonic increase in pump power with flow rate, which means that less energy is required when flow rate is reduced. Energy saving is less when H_S/H_D increases. Given a new flow rate that is less than the design flow rate, Figs. 8 - 12 and Table 4 can be used to find not only the power requirements of the five pump models but also impeller diameters needed for the new flow rate.

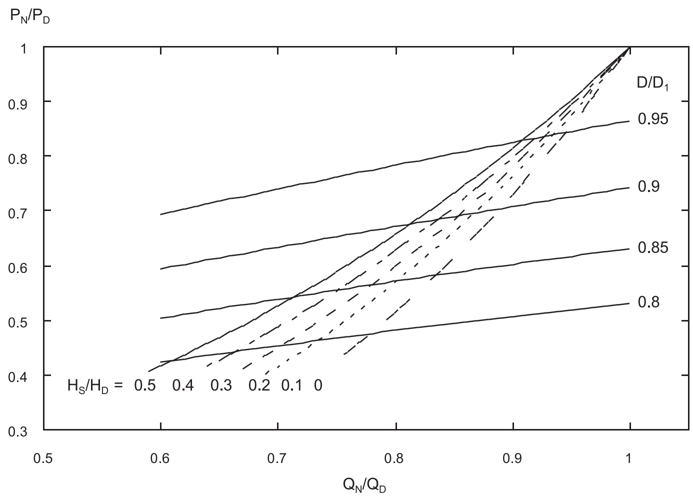

Fig. 8 Variations of power requirement of Pump #1 with flow rate at various H_S and D

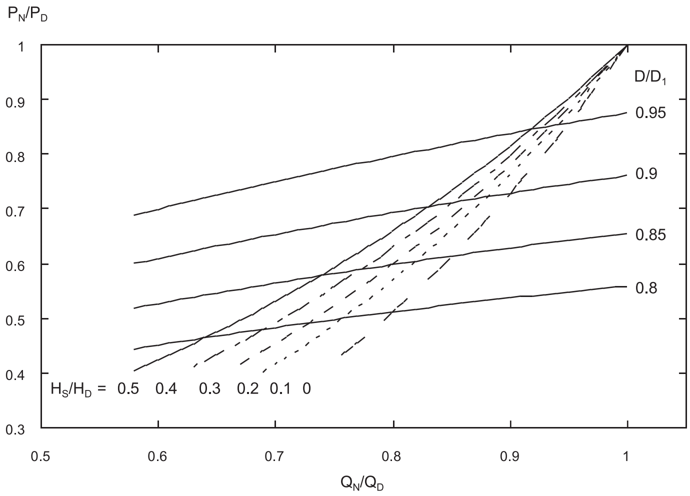

Fig. 9 Variations of power requirement of Pump #2 with flow rate at various H_S and D

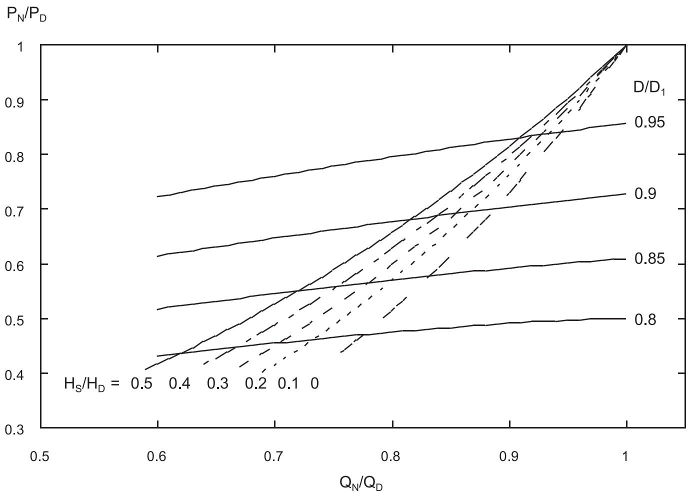

Fig. 10 Variations of power requirement of Pump #3 with flow rate at various H_S and D

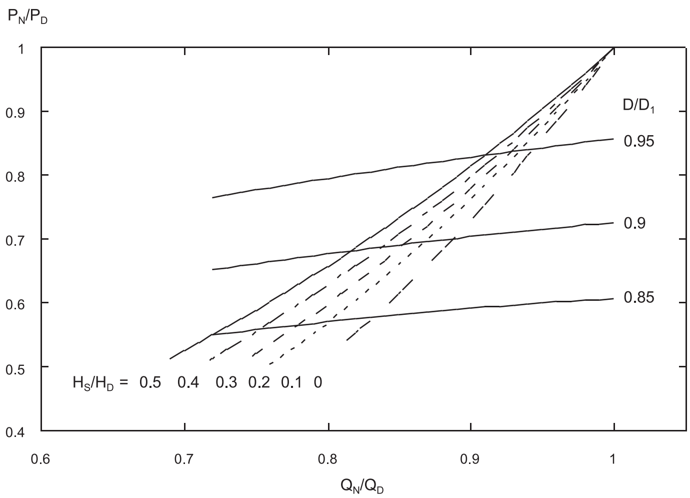

Fig. 11 Variations of power requirement of Pump #4 with flow rate at various H_S and D

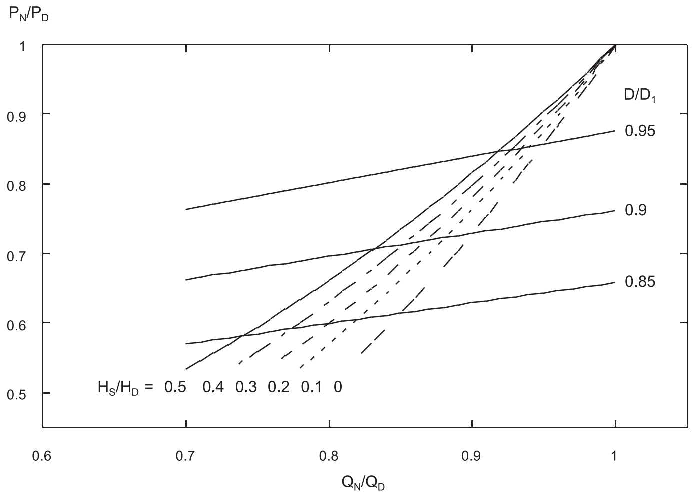

Fig. 12 Variations of power requirement of Pump #5 with flow rate at various H_S and D

It is interesting to compare results from this paper with results from the affinity laws. The widely accepted formula for computing power reduction by impeller trimming derives from Eq. (15) and (16).

$$\frac{P_1}{P_2} = \left(\frac{Q_1}{Q_2}\right)^3 \qquad \textbf{(37)}$$

Table 5 shows that, for Pump #1, power input required by the pump at reduced flow rate according to the proposed method is more than the power input according to Eq. (37). Although the difference between the two power inputs is quite small when there is no static head ($H_S/H_D = 0$), the overestimation is certainly not negligible when HS/HD is equal to or more than 0.1. Similar results for Pump #2 are shown in Table 6. In this case, the power input according to the proposed method is equal to the power input according to Eq. (37) when $H_S/H_D = 0$. However, the difference between the two power inputs increases with H_S/H_D.

Table 5 Comparison between pump powers at reduced flow rates of Pump #1 from the proposed method and Eq. (36)

Q/Q_D	Pump power (kW) according to						
	Proposed method with H_S/H_D equal to						Eq. (37)
	0	0.1	0.2	0.3	0.4	0.5	
0.75	6.34	6.79	7.26	7.74	8.24	8.76	6.28
0.80	7.66	8.07	8.48	8.90	9.34	9.79	7.62
0.85	9.17	9.50	9.84	10.19	10.55	10.91	9.14
0.90	10.86	11.11	11.36	11.61	11.87	12.12	10.85
0.95	12.76	12.90	13.03	13.17	13.31	13.45	12.76

Table 6 Comparison between pump powers at reduced flow rates of Pump #2 from the proposed method and Eq. (36)

Q/Q_D	Pump power (kW) according to						
	Proposed method with H_S/H_D equal to						Eq. (37)
	0	0.1	0.2	0.3	0.4	0.5	
0.75	54.30	58.55	62.88	67.28	71.73	76.26	54.30
0.80	65.89	69.62	73.40	77.21	81.07	84.97	65.89
0.85	79.04	82.09	85.16	88.26	91.38	94.52	79.04
0.90	93.82	96.03	98.25	100.47	102.71	104.96	93.82
0.95	110.34	111.54	112.73	113.93	115.13	116.34	110.34

6. Conclusion

A method for determining power input for a centrifugal pump having an arbitrary impeller size is presented. Pump head and efficiency are assumed to be quadratic functions of flow rate. These assumptions are supported by pump performance curves of 5 pump models supplied by 5 manufacturers. It is shown that the affinity laws that express relationships between head, efficiency, and flow rate are applicable for each pump model, but the exponent k varies from model to model. The affinity laws enable head curves and efficiency curves of a pump model having different impeller diameters to collapse into single curves. The presented method uses these curves and the assumption that the system curve is a parabola to compute power input of a pump model with trimmed impellers. It is shown that using Eq. (36) to estimate energy saving by impeller trimming will lead to an unacceptable overestimation when static head is large.

7. References

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