การวิเคราะห์กระบวนการหีบอ้อยด้วยแบบจำลองเชิงตัวเลข

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บทคัดย่อ

กระบวนการหีบอ้อยเป็นการคั้นน้ำอ้อยจากลำต้นอ้อย การหีบอ้อยด้วยลูกหีบเพียงหนึ่งลูกให้ผลที่ไม่น่าพอใจ เนื่องจากกากอ้อยที่เหลือจากการหีบยังคงมีน้ำอ้อยในปริมาณมาก ดังนั้นกระบวนการหีบอ้อยจึงมักประกอบด้วยลูกหีบอย่างน้อย 5 ลูกทำงานร่วมกันเป็นอนุกรม มีการใช้น้ำหรือน้ำอ้อยพรมลงบนกากอ้อยก่อนการหีบอ้อยในลูกหีบลูกที่สองจนถึงลูกสุดท้ายเพื่อเพิ่มปริมาณน้ำอ้อยที่หีบได้ บทความนี้นำเสนอแบบจำลองคณิตศาสตร์ที่จำลองการทำงานของกระบวนการหีบอ้อยดังกล่าว แบบจำลองนี้ได้จากสมดุลมวลของส่วนประกอบของอ้อยซึ่งได้แก่ไฟเบอร์และน้ำตาลที่ละลายในน้ำอ้อย นอกจากนี้แบบจำลองนี้ยังพิจารณาความเป็นไปได้ที่กากอ้อยและน้ำหรือน้ำอ้อยผสมกันอย่างไม่ทั่วถึง พารามิเตอร์สมรรถนะของกระบวนการหีบอ้อยได้จากข้อมูลการทำงานของลูกหีบทั้งหมด เมื่อใช้แบบจำลองนี้ศึกษาการทำงานของกระบวนการหีบอ้อย พบว่าปริมาณน้ำอ้อยและความชื้นในกากอ้อยเพิ่มขึ้นตามปริมาณน้ำพรม อย่างไรก็ตามการปรับค่าพารามิเตอร์บางตัวสามารถทำให้ได้น้ำอ้อยเพิ่มขึ้นแต่ความชื้นในกากอ้อยลดลง

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Analysis of Sugar Milling Process by Numerical Modeling

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Abstract

Sugar milling is a process in which juice is extracted from sugar cane stalks. Using only one mill to extract juice yields an unsatisfactory result because the bagasse leaving the mill still contains a lot of juice. Therefore, a milling tandem consisting of at least 5 mills is normally used in the sugar milling process. In order to maximize extraction, juice or water is mixed with input bagasse before being fed to each of the second and subsequent mills. This method is known as compound imbibition. This paper presents a model to simulate the operation of sugar mills that use compound imbibition. In addition to incorporating balances of fiber and dissolved solids, the model accounts for possibilities of non-ideal mixing between bagasse and imbibition water or juice. Operation data of the sugar mills are used to obtain performance parameters of all mills. The model is used to show how imbibition results in increases in both the extraction of dissolved solids and the moisture of bagasse leaving the milling process. However, by adjusting some performance parameters, it is possible to increase extraction and lower bagasse moisture.

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1. Introduction

Production of raw sugar from sugar cane starts with the loading of sugar cane stalks into the cane preparation unit where knives will cut cane stalks to small pieces that will be smashed into long strands of fibers by shredder. Sugar cane milling is the process in which juice is extracted from shredded sugar cane. Sugar cane mills are machinery used to accomplish this task. Juice extraction by a sugar cane mill is effected by squeezing between a floating top roll, which is driven by a power unit, and two bottom rolls known as the feed roll and the discharge roll. The objective of the milling process is to extract as much as juice consisting of dissolved solids and water as possible. Normally, one mill cannot yield satisfactory extraction; bagasse leaving the first mill still contains a lot of juice. Therefore, most sugar factories use at least 5 mills. Since bagasse that is fed to the second and subsequent mills is quite dry, and extraction of dry bagasse is ineffective, water or juice is mixed with bagasse before being fed to the mills. This mixing process is known as imbibition.

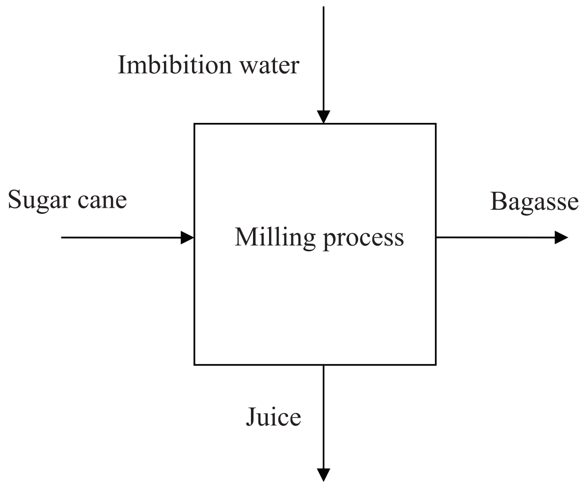

Fig. 1 Black-box model of milling process

Fig. 1 shows the black-box model of the milling process. With sufficient operating data of inputs and outputs of the milling process, bagasse and juice properties may be predicted if sugar cane properties and the amount of imbibition water are known. This approach is taken by Rein [1] and Wienese [2]. The most important drawback of this approach is the lack of the detail of the milling tandem in the milling process. Milling tandem consists of several mills connected in series. Juice recycling and interaction between mills exert great influences on outputs of the milling process. Therefore, it must be taken into account in order for the analysis of the sugar mill performance to be accurate.

This paper is concerned with the development of a milling-tandem model. This model is an improvement over models proposed by Rein [1], Wienese [2], and Loubser [3], which are simple mass balance models that do not contain performance parameters of the milling process. In the model proposed in this paper, mass balances are used to obtain performance parameters of the milling process. Once these parameters are known, inspection of the influence of a parameter on the sugar mill performance can be carried out. This model will reveal opportunities and restrictions pertaining to how to improve the performance of the sugar milling process.

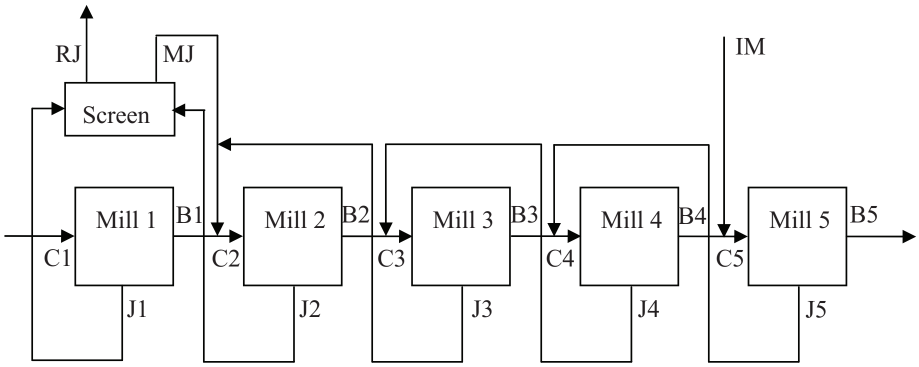

Fig. 2 Material flows in milling tandem

2. Milling Tandem

The milling tandem considered in this study consists of 5 mills in series. The imbibition scheme used in the milling tandem is compound imbibition because it is used by most sugar factories [4]. Imbibition water is mixed with bagasse leaving the next-to-last mill, the mixture is then sent to the last mill. Juice extracted from a mill is sent to mix with bagasse that is about to enter the previous mill. The milling tandem illustrated in Fig. 2 also includes a screen [4]. Prepared sugar cane (C1) flows into mill 1, which separates C1 into juice (J1) and bagasse (B1). J1 and juice from mill 2 (J2) are sent to the screen, which outputs raw juice (RJ) to be sent to subsequent processes of raw sugar manu-facturing and mixed juice (MJ) to be mixed with juice from mill 3 (J3). The mixture of B1, MJ and J3 is C2, which is the input to mill 2. The outputs of mill 2 are J2 and B2. The mixture of B2 and J4 is the input to mill 3 (C3), of which outputs are J3 and B3. Similarly, the mixture of B3 and J5 is the input to mill 4 (C4), of which outputs are J4 and B4. For mill 5, imbibitions water (IM) is mixed with B4 to become the input to mill 5 (C5). The outputs from mill 5 are B5 and J5. B5 is the final bagasse that will be used as a fuel for the boiler.

According to Fig. 2, there are 3 types of materials: sugar cane, bagasse and juice. Each material is composed of 3 components: fiber, dissolved solids and water. It should be noted that juice may contain some fiber due to imperfect juice extraction process that allows some fiber to be extracted along with dissolved solids and water. Fiber includes not only vegetable fiber but also all insoluble matters that are not fibrous such as sand and ash. Dissolved solids are sucrose and other soluble matters. Water may be divided into free water and bound water. Free water is the solvent in which sucrose and other soluble matters are dissolved. Bound water is water that forms part of cellulosic structure of sugar cane, and is not available as a solvent for sucrose and other soluble matters. Because of this, bound water is not extractable in a milling process. It is therefore more convenient to consider bound water as part of fiber. This means that water will now be referred to only free water. For the purpose of performing mass balances, meanings of symbols for flow rates at input and output of the mills are shown in Table 1. Note that symbols for water flow rates are not needed because a water flow rate is simply computed by subtracting flow rates of fiber and dissolved solids from the total flow rate.

3. Mathematical Model

In a sugar mill analysis, given quantities are m_{c1}, m_{fc1}, m_{bc1} and m_{im}. If sugar mill performance parameters are also available, other quantities in Table 1 can be determined from mass balances. First, let's consider the determination of 16 unknowns: m_r, m_m, m_{c2}, …, m_{c5}, m_{j1}, …, m_{j5}, m_{b1}, …, m_{b5}.

Table 1 Meaning of symbols

		Flow rate (t/h)		
		Total	Fiber	Dissolved Solids
Mill 1	C1	m_{c1}	m_{fc1}	m_{bc1}
	J1	m_{j1}	m_{fj1}	m_{bj1}
	B1	m_{b1}	m_{fb1}	m_{bb1}
Mill 2	C2	m_{c2}	m_{fc2}	m_{bc2}
	J2	m_{j2}	m_{fj2}	m_{bj2}
	B2	m_{b2}	m_{fb2}	m_{bb2}
Mill 3	C3	m_{c3}	m_{fc3}	m_{bc3}
	J3	m_{j3}	m_{fj3}	m_{bj3}
	B3	m_{b3}	m_{fb3}	m_{bb3}
Mill 4	C4	m_{c4}	m_{fc4}	m_{bc4}
	J4	m_{j4}	m_{fj4}	m_{bj4}
	B4	m_{b4}	m_{fb4}	m_{bb4}
Mill 5	C5	m_{c5}	m_{fc5}	m_{bc5}
	J5	m_{j5}	m_{fj5}	m_{bj5}
	B5	m_{b5}	m_{fb5}	m_{bb5}
Screen	RJ	m_r	m_{fr}	m_{br}
	MJ	m_m	m_{fm}	m_{bm}
	IM	m_{im}	-	-

The number of required equations is 16. The first equation comes from the total mass balance for the screen.

$$m_m + m_r = m_{j1} + m_{j2} \quad \textbf{(1)}$$

Define y_m as the ratio of the mixed juice flow rate to the total input flow rate into the screen.

$$m_m = y_m(m_{j1} + m_{j2}) \quad \textbf{(2)}$$

Total mass balances of all 5 mills yield the following additional equations:

$$m_{j1} + m_{b1} = m_{c1} \quad \textbf{(3)}$$
$$m_{j2} + m_{b2} = m_{c2} \quad \textbf{(4)}$$
$$m_{j3} + m_{b3} = m_{c3} \quad \textbf{(5)}$$
$$m_{j4} + m_{b4} = m_{c4} \quad \textbf{(6)}$$
$$m_{j5} + m_{b5} = m_{c5} \quad \textbf{(7)}$$
$$m_{c2} = m_{b1} + m_m + m_{j3} \quad \textbf{(8)}$$
$$m_{c3} = m_{b2} + m_{j4} \quad \textbf{(9)}$$
$$m_{c4} = m_{b3} + m_{j5} \quad \textbf{(10)}$$
$$m_{c5} = m_{b4} + m_{im} \quad \textbf{(11)}$$

The other 5 equations are related to the performances of the 5 mills. A mill consists of one floating top roll and two bottom rolls. The amount of juice extracted depends on the pressure exerted by the top roll on the bottom rolls. For a given mill pressure setting, it may be expected that the amount of extracted juice is proportional to the amount of cane input. That is, $m_{ji} = y_i m_{ci}$ (i = 1, 2, …, 5). However, such a model is inaccurate due to the presence of fiber in cane, which has an adverse effect of extracted juice. A better model must take into account the effect of fiber. As a result, it may be assumed that

$$m_{ji} = y_i(m_{ci} - m_{fci}) - (x_i - 1)y_i m_{fci} \text{ (i = 1, 2, …, 5)} \quad \textbf{(12)}$$

Once y_m, y_i x_i, and m_{fci} (i = 1, 2, …, 5) are all known, Eqs. (1) – (12) can be solved for the 16 quantities.

Next, consider the determination of 16 unknowns: m_{fr}, m_{fm}, m_{fc2}, …, m_{fc5}, m_{fj1}, …, m_{fj5}, m_{fb1}, …, m_{fb5}. Ideally, there should be no fiber in extracted juice, which will yield the simple results of $m_{fji} = 0$, and $m_{fbi} = m_{fc}$. In reality, however, a small amount of fiber can be found in extracted juice. Wienese [5] defines the fiber separation efficiency (η_{fi}) as the ratio of fiber mass in bagasse to the fiber mass in cane.

$$m_{fbi} = \eta_{fi} m_{fci} \text{ (i = 1, 2, …, 5)} \quad \textbf{(13)}$$

The fiber separation efficiency can be similarly defined for the screen.

$$m_{fm} = \eta_{fm}(m_{fj1} + m_{fj2}) \quad \textbf{(14)}$$

Additional equations come from mass balances of fiber in mills and screen.

$$m_{fj1} + m_{fb1} = m_{fc1} \quad \textbf{(15)}$$
$$m_{fj2} + m_{fb2} = m_{fc2} \quad \textbf{(16)}$$
$$m_{fj3} + m_{fb3} = m_{fb3} \quad \textbf{(17)}$$
$$m_{fj4} + m_{fb4} = m_{fb4} \quad \textbf{(18)}$$
$$m_{fj5} + m_{fb5} = m_{fb5} \quad \textbf{(19)}$$
$$m_{fc2} = m_{fb1} + m_{fm} + m_{fj3} \quad \textbf{(20)}$$
$$m_{fc3} = m_{fb2} + m_{fj4} \quad \textbf{(21)}$$
$$m_{fc4} = m_{fb3} + m_{fj5} \quad \textbf{(22)}$$
$$m_{fc5} = m_{fb4} \quad \textbf{(23)}$$
$$m_{fm} + m_{fr} = m_{fj1} + m_{fj2} \quad \textbf{(24)}$$

There are now 16 quantities (m_{br}, m_{bm}, m_{bc2}, …, m_{bc5}, m_{bj1}, …, m_{bj5}, m_{bb1}, …, m_{bb5}) left to be determined. Mass balances of dissolved solids in the mills and the screen yield

$$m_{bj1} + m_{bb1} = m_{bc1} \quad \textbf{(25)}$$
$$m_{bj2} + m_{bb2} = m_{bc2} \quad \textbf{(26)}$$
$$m_{bj3} + m_{bb3} = m_{bc3} \quad \textbf{(27)}$$
$$m_{bj4} + m_{bb4} = m_{bc4} \quad \textbf{(28)}$$

$$m_{bj5} + m_{bb5} = m_{bc5} \quad \textbf{(29)}$$
$$m_{bc2} = m_{bb1} + m_{bm} + m_{bj3} \quad \textbf{(30)}$$
$$m_{bc3} = m_{bb2} + m_{bj4} \quad \textbf{(31)}$$
$$m_{bc4} = m_{bb3} + m_{bj5} \quad \textbf{(32)}$$
$$m_{bc5} = m_{bb4} \quad \textbf{(33)}$$
$$m_{bm} + m_{br} = m_{bj1} + m_{bj2} \quad \textbf{(34)}$$

Two additional equations are based on the assumption that the concentration of dissolved solids is unchanged in mill 1 and the screen. Therefore,

$$\frac{m_{bj1}}{m_{j1} - m_{fj1}} = \frac{m_{bc1}}{m_{c1} - m_{fj1}} \quad \textbf{(35)}$$
$$\frac{m_{bm}}{m_{m} - m_{fm}} = \frac{m_{bj1} + m_{bj2}}{m_{j1} - m_{fj1} + m_{j2} - m_{fj2}} \quad \textbf{(36)}$$

Mills 2, 3, 4 and 5 differ from mill 1 due to imbibition. The input to each of these mills is a mixture of recycled juice or water and the bagasse from the previous mill. For example, the input to mill 2 is a mixture of B1, MJ and J3. The assumption that the concentration of dissolved solids in the extracted juice from each of these mills is the same as the concentration of dissolved solids in the input may not be true because the input may not be homogeneous due to incomplete mixing between recycled juice or water and the bagasse. Rein [4] argues that the imbibition juice is more easily extracted than the juice already imbedded in the bagasse. This results in the extracted juice having lower concentration of dissolved solids than the juice imbedded in the bagasse [6]. In order to model non-ideal mixing, Wienese [5] defines imbibition efficiency as the ratio of concentration of dissolved solids in the juice extracted from the mill to the concentration of dissolved solids in the input to the mill. According to this definition, the imbibition efficiency of mill i is

$$\eta_i = \frac{m_{bji}/(m_{ji} - m_{fji})}{m_{bci}/(m_{ci} - m_{fci})} \quad \textbf{(37)}$$

If η_i (i = 2, 3, 4, 5) are known, Eqs. (25) – (37) can be solved for the 16 unknowns.

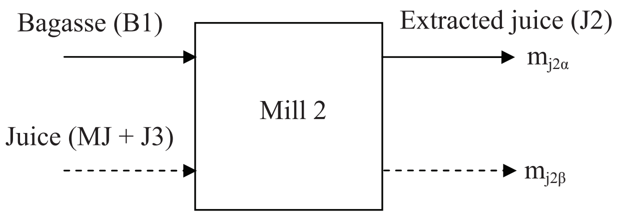

Fig. 3 Model of juice extraction in mill 2

It is interesting to consider a model of non-ideal mixing in mills 2, 3, 4 and 5. A model of the juice extraction in mill 2 is illustrated in Fig. 3. It can be seen that the total extracted juice (m_{j2}) is the sum of juice extracted from B1 ($m_{j2\alpha}$) and juice extracted from MJ and J3 ($m_{j2\beta}$). Expressions of $m_{j2\alpha}$ and $m_{j2\beta}$ are assumed to be

$$m_{j2\alpha} = y_2(m_{b1} - m_{fb1}) - (x_2 - 1)y_2 m_{fb1} \quad \textbf{(38)}$$
$$m_{j2\beta} = y_2(m_m + m_{j3} - m_{fm} - m_{fj3}) - (x_2 - 1)y_2(m_{fm} + m_{fj3}) \quad \textbf{(39)}$$

The concentration of dissolved solids in J2 is the weighted average of the concentrations of dissolved solids in juice extracted from B1 and MJ + J3. Therefore,

$$\left(\frac{m_{bj2}}{m_{b2}-m_{fb2}}\right)m_{j2} = \left(\frac{m_{bb1}}{m_{b1}-m_{fb1}}\right)m_{j2\alpha} + \left(\frac{m_{bm}+m_{bj3}}{m_{m}-m_{fm}+m_{j3}-m_{fj3}}\right)m_{j2\beta} \quad \textbf{(40)}$$

Another expression of m_{bj2} comes from Eq. (37).

$$\left(\frac{m_{bj2}}{m_{b2}-m_{fb2}}\right)m_{j2} = \eta_2\left(\frac{m_{j2}-m_{fj2}}{m_{b1}-m_{fb1}+m_{m}-m_{fm}+m_{j3}-m_{fj3}}\right)\left(m_{bb1}+m_{bm}+m_{bj3}\right) \quad \textbf{(41)}$$

Equations (38) – (41) lead to the following functional relationship between x_2 and η_2.

$$\eta_2 = \frac{1-\left(\frac{x_2-1}{m_{bc2}}\right)\left[\left(\frac{m_{bb1}m_{fb1}}{m_{b1}-m_{fb1}}\right)+\left(\frac{\left(m_{bm}+m_{bj3}\right)\left(m_{fm}+m_{fj3}\right)}{m_{m}+m_{j3}-m_{fm}-m_{fj3}}\right)\right]}{1-\left(\frac{x_2-1}{m_{c2}-m_{fc2}}\right)m_{fc2}} \quad \textbf{(42)}$$

Similar results can be obtained for mills 3, 4 and 5.

$$\eta_3 = \frac{1-\left(\frac{x_3-1}{m_{bc3}}\right)\left[\left(\frac{m_{bb2}m_{fb2}}{m_{b2}-m_{fb2}}\right)+\left(\frac{m_{bj4}m_{fj4}}{m_{j4}-m_{fj4}}\right)\right]}{1-\left(\frac{x_3-1}{m_{c3}-m_{fc3}}\right)m_{fc3}} \quad \textbf{(43)}$$

$$\eta_4 = \frac{1-\left(\frac{x_4-1}{m_{bc4}}\right)\left[\left(\frac{m_{bb3}m_{fb3}}{m_{b3}-m_{fb3}}\right)+\left(\frac{m_{bj5}m_{fj5}}{m_{j5}-m_{fj5}}\right)\right]}{1-\left(\frac{x_4-1}{m_{c4}-m_{fc4}}\right)m_{fc4}} \quad \textbf{(44)}$$

$$\eta_5 = \frac{1-\left(\frac{x_5-1}{m_{b4}-m_{fb4}}\right)m_{fb4}}{1-\left(\frac{x_5-1}{m_{c5}-m_{fb4}}\right)m_{fc4}} \quad \textbf{(45)}$$

According to Eqs. (42) – (45), ideal mixing in mills 2, 3, 4 and 5 occurs when $x_i = 1$, which leads to $\eta_i = 1$ (i = 2, 3, 4, 5). The fact that $\eta_i < 1$ means that $x_i > 1$. This means that, in mill 2, juice extraction from B1 is more difficult than juice extraction from MJ + J3. As a result, less juice is extracted from B1 due to non-ideal mixing between B1 and MJ + J3. Let s_2 be the ratio of the amount of juice extracted from B1 when mixing between B1 and MJ + J3 is non-ideal to the amount of juice extracted from B1 when mixing between B1 and MJ + J3 is ideal.

$$s_2 = \frac{m_{b1} - x_2 m_{fb1}}{m_{b1} - m_{fb1}} \quad \textbf{(46)}$$

Similar expressions can be obtained for s_3, s_4 and s_5.

$$s_3 = \frac{m_{b2} - x_3 m_{fb2}}{m_{b2} - m_{fb2}} \quad \textbf{(47)}$$

$$s_4 = \frac{m_{b3} - x_4 m_{fb3}}{m_{b3} - m_{fb3}} \quad \textbf{(48)}$$

$$s_5 = \frac{m_{b4} - x_5 m_{fb4}}{m_{b4} - m_{fb4}} \quad \textbf{(49)}$$

It can be seen that $s_i < 1$ in an actual sugar mill due to non-deal mixing between bagasse and imbibition.

4. Mill Performance Parameters

There are a total of 17 performance parameters required in the mathematical model presented in Section 3. They are y_m, η_{fm}, x_i, y_i and η_{fi}, (i = 1, 2, …, 5). However, the model does not provide a method for determining x_1. Hence, its value must be assumed. This leaves 16 parameters to be determined. Measurements of the sugar mills are required to find values of these parameters. In addition to 4 given quantities (m_{c1}, m_{fc1}, m_{bc1} and m_{im}), the 16 quantities needed to be measured are m_{j1}, m_{fj1}, m_{j2}, m_{fj2}, m_{bj2}, m_{j3}, m_{fj3}, m_{bj3}, m_{j4}, m_{fj4}, m_{bj4}, m_{j5}, m_{fj5}, m_{bj5}, m_m and m_{fm}. The procedure for determining the mill performance parameters from these quantities is as follows.

1. Compute m_{bj1} and m_{bm} from Eqs. (35) and (36).

$$m_{bj1} = \left(\frac{m_{bc1}}{m_{c1} - m_{fc1}}\right)\left(m_{j1} - m_{fj1}\right) \quad \textbf{(50)}$$

$$m_{bm} = \left(\frac{m_{bj1} + m_{bj2}}{m_{j1} - m_{fj1} + m_{j2} - m_{fj2}}\right)\left(m_m - m_{fm}\right) \quad \textbf{(51)}$$

2. Compute m_{bi} (i = 1, 2, .., 5) from Eqs. (3) – (7).

$$m_{bi} = m_{ci} - m_{ji} \quad \textbf{(52)}$$

3. Compute m_{fbi} (i = 1, 2, .., 5) from Eqs. (15) – (23).

$$m_{fb1} = m_{fc1} - m_{fj1} \quad \textbf{(53)}$$

$$m_{fb2} = m_{fb1} + m_{fm} + m_{fj3} - m_{fj2} \quad \textbf{(54)}$$

$$m_{fb3} = m_{fb2} + m_{fj4} - m_{fj3} \quad \textbf{(55)}$$

$$m_{fb4} = m_{fb3} + m_{fj5} - m_{fj4} \quad \textbf{(56)}$$

$$m_{fb5} = m_{fb4} - m_{fj5} \quad \textbf{(57)}$$

4. Compute m_{bbi} (i = 1, 2, .., 5) from Eqs. (25) – (33).

$$m_{bb1} = m_{bc1} - m_{bj1} \quad \textbf{(58)}$$

$$m_{bb2} = m_{bb1} + m_{bm} + m_{bj3} - m_{bj2} \quad \textbf{(59)}$$

$$m_{bb3} = m_{bb2} + m_{bj4} - m_{bj3} \quad \textbf{(60)}$$

$$m_{bb4} = m_{bb3} + m_{bj5} - m_{bj4} \quad \textbf{(61)}$$

$$m_{bb5} = m_{bb4} - m_{bj5} \quad \textbf{(62)}$$

5. Compute η_{fm} and η_{fi} (i = 1, 2, …, 5) from Eqs. (13) and (14).

$$\eta_{fm} = \frac{m_{fm}}{m_{fj1} + m_{fj2}} \tag{63}$$

$$\eta_{fi} = \frac{m_{fbi}}{m_{fci}} \quad (i = 1, 2, ..., 5) \tag{64}$$

6. Compute η_i (i = 2, 3, 4, 5) from Eq. (37).
7. Compute x_i (i = 2, 3, 4, 5) from Eqs. (42) – (45).

$$x_2 = 1 + \frac{(1-\eta_2)}{\left[\frac{m_{bb1} m_{fb1}}{(m_{b1} - m_{fb1}) m_{bc2}} + \frac{(m_{bm} + m_{bj3})(m_{fm} + m_{fj3})}{(m_m + m_{j3} - m_{fm} - m_{fj3}) m_{bc2}} - \frac{\eta_2 m_{fc2}}{(m_{c2} - m_{fc2})}\right]} \tag{65}$$

$$x_3 = 1 + \frac{(1-\eta_3)}{\left[\frac{m_{bb2} m_{fb2}}{(m_{b2} - m_{fb2}) m_{bc3}} + \frac{m_{bj4} m_{fj4}}{(m_{j4} - m_{fj4}) m_{bc3}} - \frac{\eta_3 m_{fc3}}{(m_{c3} - m_{fc3})}\right]} \tag{66}$$

$$x_4 = 1 + \frac{(1-\eta_4)}{\left[\frac{m_{bb3} m_{fb3}}{(m_{b3} - m_{fb3}) m_{bc4}} + \frac{m_{bj5} m_{fj5}}{(m_{j5} - m_{fj5}) m_{bc4}} - \frac{\eta_4 m_{fc4}}{(m_{c4} - m_{fc4})}\right]} \tag{67}$$

$$x_5 = 1 + \frac{(1-\eta_5)}{m_{fb4}\left[\frac{1}{(m_{b4} - m_{fb4})} - \frac{\eta_5}{(m_{c5} - m_{fb4})}\right]} \tag{68}$$

8. Finally, compute y_m and y_i (i = 1, 2, …, 5) from Eqs. (2) and (12).

$$y_m = \frac{m_m}{m_{j1} + m_{j2}} \tag{69}$$

$$y_i = \frac{m_{ji}}{m_{ci} - x_i m_{fci}} \tag{70}$$

It is appropriate to end this section by using hypothetical mill data shown in Table 2 to compute mill performance parameters that will later be used to analyze the performance of the milling tandem. The above procedure yield, for i = 1, 2, … 5, $\eta_{fm} = \eta_{fi} = 0.9$, $y_m = 0.15$, $y_i = 0.95$, $x_2 = 1.702$, $x_3 = 1.531$, $x_4 = 1.415$, and $x_5 = 1.345$. Since the model does not specify how to compute x_1, its value may be approximated using extrapolation from x_2, x_3, x_4 and x_5. The assigned value of x_1 is 2.000.

5. Results and Discussion

The performance of the hypothetical sugar mill can now be analyzed. In general, the function of a sugar mill is to extract as much sucrose as possible from the input sugar cane. Since it is possible to determine sucrose extraction from the extraction of dissolved solids [7], it is sufficient to use the extraction of dissolved solids, defined as the ratio of the dissolved solids in raw juice to the dissolved solids in the input sugar cane, as a measure the performance of sugar mills.

$$\varepsilon = \frac{m_{br}}{m_{bc}} \qquad \textbf{(71)}$$

Another measure of the performance of sugar mills is the moisture of final bagasse, which is the bagasse leaving mill 5. Bagasse moisture is defined as the total water content in bagasse divided by the total mass of bagasse. It was previously mentioned in Section 2 that the water content is the sum of free water and bound water. For final bagasse, the free water content is $m_{b5} - m_{fb5} - m_{bb5}$. According to Rein [4], bound water accounts for 27% of fiber. Therefore, the moisture of final bagasse is

$$\omega = \frac{\left(m_{b5} - 0.73\, m_{fb5} - m_{bb5}\right)}{m_{b5}} \qquad \textbf{(72)}$$

It is desirable for the moisture of final bagasse to be small because it is used as a fuel for the boiler. High moisture in bagasse not only makes complete combustion more difficult but also reduces available useful heat produced by the boiler.

Table 2 Hypothetical mill data.

		Flow rate (t/h)		
		Total	Fiber	Dissolved Solids
Mill 1	C1	100.00	19.00	16.00
	J1	58.90	1.90	11.26
	B1	41.10	17.10	4.74
Mill 2	C2	111.23	23.22	9.57
	J2	68.14	2.32	6.31
	B2	43.10	20.90	3.26
Mill 3	C3	89.28	23.19	4.58
	J3	51.08	2.32	2.65
	B3	38.20	20.87	1.94
Mill 4	C4	81.06	22.94	2.45
	J4	46.18	2.29	1.32
	B4	34.89	20.64	1.12
Mill 5	C5	72.89	20.64	1.12
	J5	42.89	2.06	0.51
	B5	30.02	18.58	0.62
Screen	RJ	107.98	0.42	15.39
	MJ	19.06	3.80	2.18
	IM	38.00	-	-

Although there are many factors that affect the performance of the sugar mill, only 3 factors are considered here. They are the amount of imbibition water, the pressure settings of the mills, and the degree of mixing between bagasse and imbibition juice or water.

5.1 Amount of Imbibition Water

Assume that performance parameters of the sugar mill as shown in Table 2 do not change with the amount of imbibition water. Fig. 4 shows variations of ε and ϖ with the amount of imbibition water, which varies for 100% to 500% of the fiber mass in the input cane. Figure 4 confirms the well-known fact that increasing the amount of imbibition water increases both extraction and final bagasse moisture. However, the rates of increase of ε and ϖ are not identical. When the amount of imbibition water is small, ε increases more rapidly than ϖ. But then the amount of imbibition water is large, ε increases less rapidly than ϖ. This suggests that there should be an optimum amount of imbibition water.

5.2 Pressure Settings of the Mills

Pressure exerted by the top roll of each mill is designed so that juice extraction by the mill is optimum. Although higher pressure can yield more juice extraction, it may be beyond the capacity of the mill to exert such a high pressure without compromising maintenance requirements.
Nevertheless, it is interesting to see what can be achieved by higher pressure settings. Fig. 5 shows that a higher value of y_i (i = 1, 2, …, 5), corresponding to higher pressure settings of all 5 mills, results in higher ε and lower ϖ.

5.3 Degree of Mixing between Bagasse and Juice

Imbibition juice or water is normally poured onto the bed of bagasse at the exit from a mill so that there is enough time for mixing between bagasse and imbibition before the mixture enters the next mill. However, mixing also depends on the thickness of bagasse bed. A thick bed may make it difficult for imbibition to penetrate to lower layers of bagasse, resulting in most imbibition being concentrated in top layers. Figure 6 is plotted under the assumption that the degrees of mixing of mills 2, 3, 4 and 5 are equal and represented by s_i. As expected, better mixing increases ε and decreases ϖ.

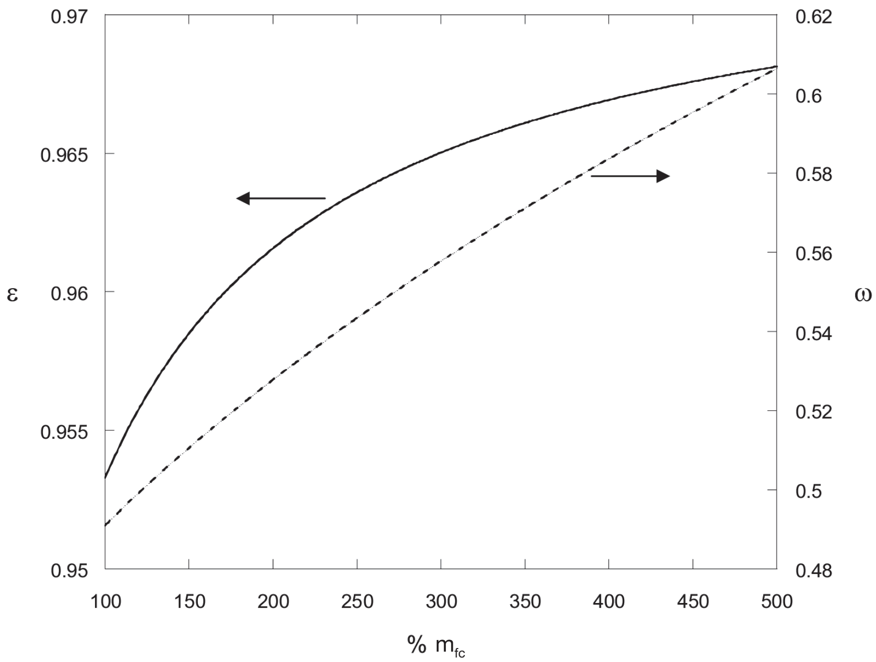

Fig. 4 Variations of extraction and final bagasse moisture with imbibition water expressed as percentage of input fiber mass.

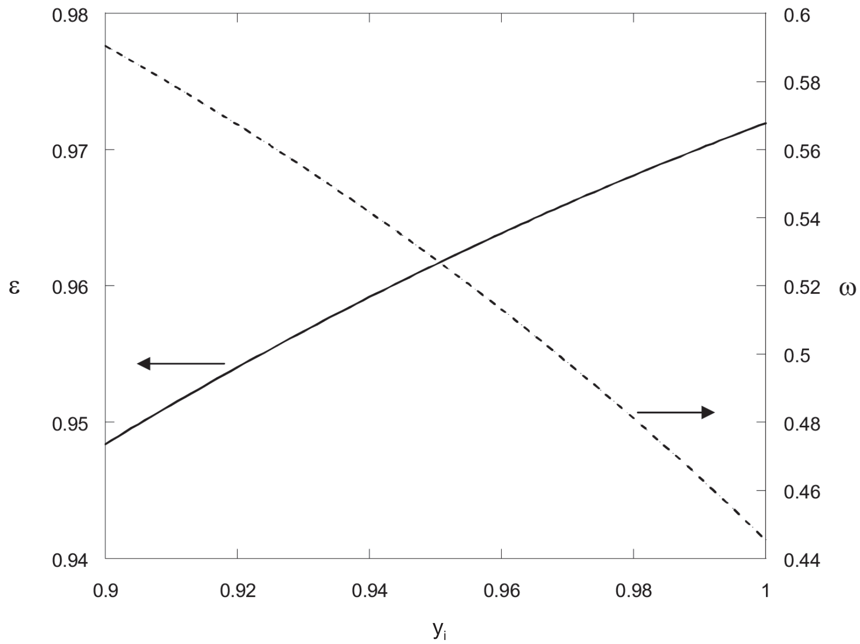

Fig. 5 Variations of extraction and final bagasse moisture with pressure settings, represented by y_i.

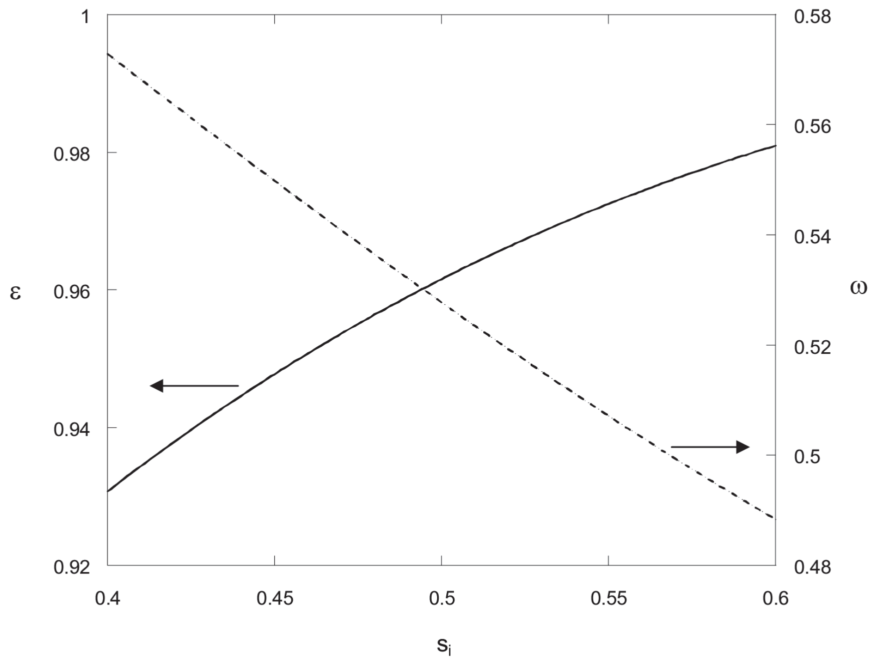

Fig. 6 Variations of extraction and final bagasse moisture with the degree of mixing between bagasse and imbibition juice or water.

6. Conclusion

A model to simulate sugar milling process with compound imbibition is presented. Required input data for the model are flow rates of sugar cane, fiber, dissolved solids, and imbibition water. Required performance parameters are computed from measurements of flow rates and contents of juice leaving the sugar mills. The model is capable of determining flow rates and contents of both bagasse and juice at the exit of each mill. It is therefore capable of determining the extraction and the moisture of the final bagasse. By using hypothetical mill data, it is shown that the effects of increasing the amount of imbibition water are increases in both extraction and final bagasse moisture. Increased extraction and decreased final bagasse moisture can be effected by adjusting mill settings in order to yield more juice extraction or by increasing the degree of mixing between bagasse and imbibition juice or water. Although only a milling tandem consisting of 5 mills is considered in this paper, it should be noted that the proposed model is still applicable if there are more than 5 mills in a milling tandem.

7. References

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