การพัฒนาสูตรสำหรับคำนวณกำลังงานของเครื่องสูบแบบหอยโข่งที่ปรับความเร็วรอบได้

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บทคัดย่อ

การควบคุมความเร็วรอบของเครื่องสูบแบบหอยโข่งเป็นวิธีอนุรักษ์พลังงานที่มีประสิทธิผล วิธีนี้ต้องใช้เงินลงทุนจำนวนมากซึ่งหมายความว่า พลังงานที่ประหยัดได้ต้องคุ้มค่ากับเงินลงทุน การประเมินพลังงานที่ประหยัดได้ด้วยการควบคุมความเร็วรอบที่นิยมใช้กันคือ การใช้กฎความคล้ายซึ่งระบุว่า พลังงานที่ใช้เดินเครื่องสูบแปรผันตามอัตราการไหลยกกำลังสาม อย่างไรก็ตาม การใช้กฎความคล้ายอย่างตรงไปตรงมาเช่นนี้อาจนำไปสู่การประเมินการประหยัดพลังงานที่มากเกินความเป็นจริงถ้าเครื่องสูบต้องส่งผ่านของเหลวในระบบที่มีทั้งความเสียดทานในท่อและมีเฮดสถิต แต่วิศวกรจำนวนมากก็ยังคงใช้วิธีนี้เนื่องจากไม่มีวิธีอื่นที่สะดวกต่อการใช้งาน บทความนี้แสดงให้เห็นว่า ถ้ามีข้อมูลของเครื่องสูบและระบบการไหลที่เพียงพอ การสร้างสูตรคำนวณที่ประมาณการประหยัดพลังงานได้แม่นยำกว่าสามารถกระทำได้

คำสำคัญ : การอนุรักษ์พลังงาน / ประสิทธิภาพการใช้พลังงาน / กฎความคล้าย

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Development of Formula for Computing Power Input of Variable-speed Centrifugal Pumps

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Abstract

Variable-speed control of centrifugal pumps is an effective energy conservation method. This method requires substantial investment, which must be justified by energy saving. A popular method for energy saving estimation is the use of the well-known affinity laws, which imply that power input varies with the cube of the flow rate. However, this straightforward application of the affinity laws may lead to an overestimation of energy saving if a pump must deliver a fluid against not only the frictional resistance in the piping system but also static head or differential pressure. Nevertheless, many practicing engineers still rely on the affinity laws due to the lack of alternative formula. This paper shows that a formula that provides better estimation of energy saving of a variable-speed centrifugal pump may be derived provided that sufficient details of pump and system characteristics are known.

Keywords : Energy Conservation / Energy Efficiency / Affinity Laws

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1. Introduction

Centrifugal pumps are common fluid machinery found in virtually all factories and buildings. Energy required to operate pumps accounts for a significant fraction of total energy use by a factory or a building. Therefore, much attention has been paid to methods for energy conservation in pumping systems. Kaya et al. [1] suggested many energy saving opportunities in operation of pumps. One of their suggestions is the installation of variable-speed control. It is a standard practice to specify an oversized pump during the design stage to allow for either future expansion or unforeseen losses. An oversized pump may deliver too much flow rate, and necessitate the use of a throttle valve to reduce flow rate. Such a practice results in energy inefficiency. Variable-speed control is a more energy efficient way to reduce flow rate. Studying the optimum sizing and design of a pumping station unit in a hybrid wind-hydro plant, Anagnostopoulos and Papantonis [2] found that the use of a variable-speed pump constituted the most effective and profitable solution, and its superiority was more pronounced for less dispersed wind power potential. Garibotti [3] analyzed the use of variable-speed control of centrifugal pump in brine blow-down service of a desalination plant, and found that, compared with throttling control, variable-speed control offered advantages in not only reduced power consumption but also more stabilized control.

A convenient way to estimate energy saving to be expected from variable-speed pumps to use the affinity laws, represented by the following equations.

$$\frac{Q_1}{Q_2} = \frac{N_1}{N_2} \quad \textbf{(1)}$$

$$\frac{H_1}{H_2} = \left(\frac{N_1}{N_2}\right)^2 \quad \textbf{(2)}$$

$$\frac{P_1}{P_2} = \left(\frac{N_1}{N_2}\right)^3 \quad \textbf{(3)}$$

where Q is flow rate, N is pump speed, H is head, and P is power input. Equations (1) and (3) may be combined into

$$\frac{P_1}{P_2} = \left(\frac{Q_1}{Q_2}\right)^3 \quad \textbf{(4)}$$

It has been recognized that using Eq. (4) results in a correct determination of energy saving from the variable-speed control of a centrifugal pump only when the pump delivers fluid against frictional resistance in the piping system. When there is a static head or differential pressure to overcome, using Eq. (4) will result in overestimation of energy saving [4]. Since the installation of variable-speed control for pumps requires a substantial investment, it is imprudent to rely on Eq. (4) in making the investment decision. There have been several attempts at providing a better estimate of the power input of a variable-speed pump [5-10]. Despite all these attempts, however, Eq. (4) remains the popular among practicing engineers.

The main objective of this paper is to develop formulas for computing power input of variable-speed pumps. It will be shown that no simple functional relationship between P_1/P_2 and Q_1/Q_2 like Eq. (4) exists that can be applied to all variable-speed pumps because centrifugal pumps are available in a wide variety of sizes, designs, and capacities, and they operate in different systems. However, it will be shown that, if pump performance data are given, and sufficient information about the pumping system is known, it is possible to develop such a formula.

2. Energy Saving of Variable-speed Pumps

The determination of energy saving of a variable-speed pump is equivalent to the determination of the power input of the pump. There have been several attempts to provide formulas or methods of determining a more realistic estimate of energy saving resulting from using variable-speed control of pumps. Lee [5] proposed that the projected energy saving might be obtained by decreasing the exponent in Eq. (4) from 3 to 2.5. He also measured energy savings of 9 case studies of variable-speed pumps. The discrepancies between projected savings and measured energy savings of these case studies indicate, however, that changing the exponent of Eq. (4) to a fixed value does not yield a good estimate of energy savings.

In an alternative approach taken by Vaillencourt [6], Eq. (1) is replaced with

$$\frac{N_1}{N_d} = \left(1-\sqrt{\frac{H_s}{H_d}}\right)\frac{Q_1}{Q_d}+\sqrt{\frac{H_s}{H_d}} \quad \textbf{(5)}$$

where N_2, H_2, and P_2 in Eqs. (2) and (3) are replaced with N_d, H_d, and P_d, which are the design pump speed, design head, and design power input, respectively. Point 1 is the new operating point with reduced flow rate Q_1. The static head may be interpreted as the minimum head or the static head (H_s) that must be delivered by the pump having the speed of N_{min}. Although this method is simple to use, its correctness is questionable because Vaillencourt [6] did not consider the possibility that pump efficiency at point 1 might be less than the design pump efficiency.

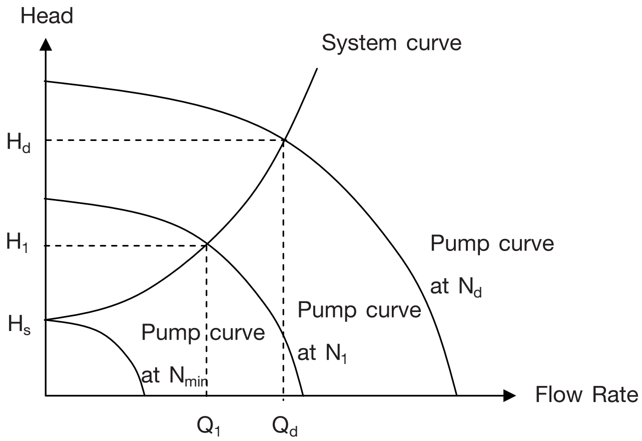

Fig. 1 Demonstration that the existence of static head (H_S) requires the pump speed to be at least N_{min}.

When there is static head in pumping system, the system curve deviates from the affinity laws curve as illustrated in Fig. 2. Therefore, the affinity laws are not applicable between (Q_1, H_1) and (Q_d, H_d). Knowledge of pump curve at N_d will allow the determination of (Q_2, H_2), which is at the intersection between the affinity laws curve and the pump curve. Note that $N_2 = N_d$ because (Q_2, H_2) is on the pump curve. Since Q_1 is given, N_1 can be determined from Eq. (1), and the power input at (Q_1, H_1) can be determined from Eq. (3). Bernier and Lemire [7] used this computational procedure to determine how power inputs of two pumps, one with a steep characteristic curve and the other with a flat characteristic curve, varied with flow rate, static head, and pump efficiency. They found that the ratio of pump power input at reduced speed to pump power input at the design speed was almost independent of pump efficiency if the flow rate is more than 70% of the design flow rate. This ratio is given as a polynomial function of flow rate and static head for the pump with a flat characteristic curve. Carlson [8] used this computational procedure to compare average electrical energy requirements of a fixed-speed pump and a variable-speed pump, and found that the energy saving depended on the process configuration. Fu et al. [9] used this computational procedure to find energy savings of variable-speed control of 4 commercial pumps. Average energy savings at various flow rates and static heads are shown in a table. They recommended using this table for evaluating energy savings of other variable-speed pumps.

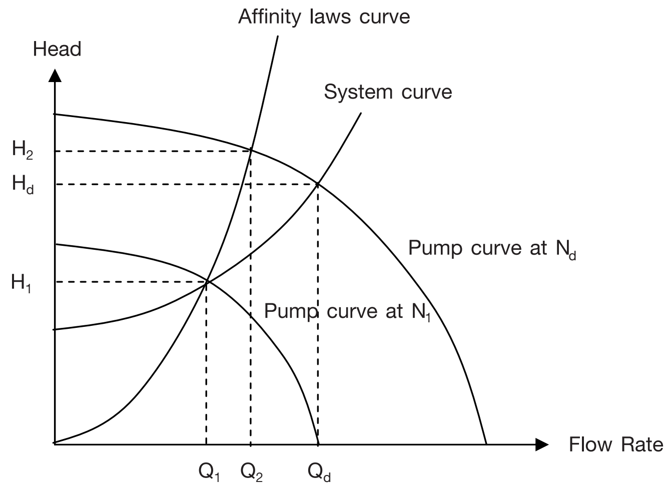

Fig. 2 Illustration of a method to find the pump speed (N_1) to deliver the required fluid flow rate (Q_1), given the system curve, the pump performance curve, and the design flow rate

Recently, Chantasiriwan [10] developed another method for determining the power input of variable-speed pumps. This method is based on the suggestion by Ulanicki et al. [11] that pump characteristics curves at different speeds obey the affinity laws. Therefore, it is possible to construct pump head function and efficiency functions that depend on flow rates and pump speed. Equating the system head equation and the pump head equation yields an equation that can be solved for pump speed provided that flow rate is given. Subsequently, the efficiency at known flow rate and pump speed can be determined. Pump power input can be computed from pump head, flow rate, and efficiency. In addition, Chantasiriwan [10] also proposed the characteristics of the theoretical variable-speed pump, and changed the exponent of Eq. (4) from 3 to α, which is a function of flow rate and static head. Variations of α with flow rate and static head were plotted in a chart that can be used to estimate energy saving of variable-speed pumps. However, there is no demonstration that performance curves of commercial pumps resemble those of the theoretical pump.

3. Theoretical Pump Performance Curves

It may be assumed that a theoretical centrifugal pump is designed to operate at the design head (H_d) and the design flow rate (Q_d) and, where the efficiency (η_d) is maximum. Ulanicki et al. [11] suggested that pump head and efficiency may be taken to be polynomial functions of flow rate as follows:

$$H = aQ^2 + bQ + c \quad \textbf{(6)}$$

$$\eta = dQ^3 + eQ^2 + fQ + g \quad \textbf{(7)}$$

Values of unknown coefficients in Eq. (6) are determined from (1) known values of H_d and Q_d, (2) known values of the maximum head (H_m) and the corresponding flow rate (Q_m), and (3) the zero slope of H function at $Q = Q_m$. Values of unknown parameters in Eq. (7) are determined from (1) known values of η_d and Q_d, (2) the zero slope of η function at $Q = Q_d$, (3) zero value of η at $Q = 0$, and (4) zero value of η at $Q = Q_m$. These conditions are used to determined the parameters in Eqs. (6) and (7). The results are the following expressions of the head and efficiency functions:

$$H = \frac{H_m(Q_0 - Q)(Q_0 + Q - 2Q_m)}{(Q_0 - Q_m)^2} \quad \textbf{(8)}$$

$$\eta = \frac{\eta_d Q(Q_0 - Q)(2Q_dQ - Q_0Q + 2Q_dQ_0 - 3Q_d^2)}{Q_d^2(Q_0 - Q_d)^2} \quad \textbf{(9)}$$

In order to reduce the number of parameters affecting the performance curves, it is necessary to convert Eqs. (8) and (9) to dimensionless equations. Let's define dimensionless flow rate and head as

$$q = \frac{Q}{Q_d} \quad \textbf{(10)}$$

$$h = \frac{H}{H_d} \quad \textbf{(11)}$$

Figure 3 shows curves of model pump head and efficiency as functions of flow rate. Equations (10) and (11) become

$$h = \frac{h_m(q_0 - q)(q_0 + q - 2q_m)}{(q_0 - q_m)^2} \quad \textbf{(12)}$$

$$\eta = \frac{\eta_d q(q_0 - q)(2q - q_0q + 2q_0 - 3)}{(q_0 - 1)^2} \quad \textbf{(13)}$$

Equations (12) and (13) require the value of Q_0, which may be computed from Eq. (12). The expression for Q_0 is

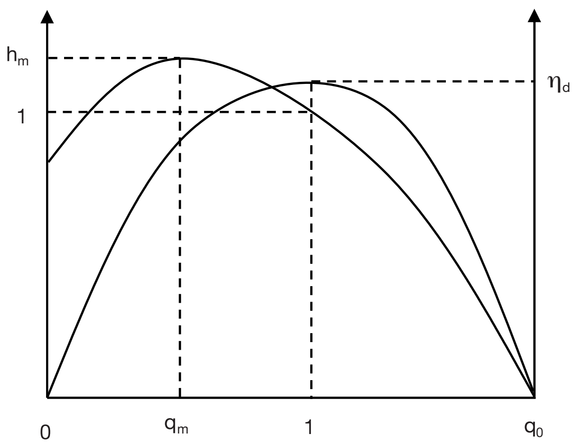

Fig. 3 Head curve and efficiency curves of the theoretical pump

$$Q_0 = (Q_d - Q_m)\sqrt{\frac{H_m}{H_m - H_d}} + Q_m \quad \textbf{(14)}$$

Once Q, H, and η are known, the required pump power is computed from

$$P = \frac{\gamma QH}{\eta} \quad \textbf{(15)}$$

where γ is specific gravity. Furthermore, define dimensionless pump power as

$$p = \frac{P}{P_d} \quad \textbf{(16)}$$

where P_d is the pump power at the design operation point:

$$P_d = \frac{\gamma Q_d H_d}{\eta_d} \quad \textbf{(17)}$$

The resulting expression for dimensionless power is

$$p = \frac{h_m(q_0 + q - 2q_m)(q_0 - 1)^2}{(2q - q_0 q + 2q_0 - 3)(1 - q_m)^2} \quad \textbf{(18)}$$

If Eqs. (12) and (18) are the pump characteristics at the speed of N, which differs from the design pump speed (N_d), pump characteristics at the speed of N_2 are given by [11].

$$\frac{h}{n^2} = \frac{h_m(q_0 - q')(q_0 + q' - 2q_m)}{(q_0 - q_m)^2} \quad \textbf{(19)}$$

$$\frac{p}{n^3} = \frac{h_m(q_0 + q' - 2q_m)(q_0 - 1)^2}{(2q' - q_0 q' + 2q_0 - 3)(1 - q_m)^2 \eta_d} \quad \textbf{(20)}$$

where $n = N/N_d$, and $q' = q/n$.

4. Commercial Pump Performance Curves

Pump manufacturers have tested their pumps extensively, and used them to construct pump performance curves. Complete pump performance curves consist of head curve, power curve, and efficiency curve showing variations of pump head, power, and efficiency, respectively, with flow rate. Most manufacturers provide only head curves for different impeller sizes, together with superimposed iso-efficiency lines. Power curves are not usually given because they can be easily drawn from the head and efficiency curves. The following 5 commercial pumps are chosen for this study because the manufacturers make their pump performance curves available for download.

- Apex Pumps (http://www.apexpumps.com)
- Aurora (http://www.aurorapump.com)
- Bell & Gossett (http://www.bellgossett.com)
- Goulds Pumps (http://www.gouldspumps.com)
- GEA Tuchenhagen (http://www.tuchenhagen.com)

Table 1 gives details of the 5 centrifugal pumps.

Table 1 Details of 5 commercial pumps chosen for this study

Pump Number	Manufacturer	Model	Design Speed (rpm)	Impeller Size (m)
1	Apex	TD 100-200	2965	0.214
2	Aurora	340 3× 4× 9B	2880	0.229
3	Bell & Gossett	4× 6× 10M HSC	3565	0.305
4	Goulds	3656/3756 S	3500	0.171
5	Tuchenhagen	TP 1020-4	2900	0.130

Figure 4 shows a plot of typical pump performance curves that can be obtained from pump manufacturers listed in Table 1. It can be seen that there are different head curves corresponding to different impeller diameters (D_1 – D_4). Each plot corresponds to a specific pump speed. The nominal speed is about 2900 rpm for a pump run by 50-Hz motor, and the nominal speed is about 3500 rpm for a pump run by 60-Hz motor. For each pump model, manufacturers usually provide at least 2 plots corresponding to the nominal speed and half the nominal speed.

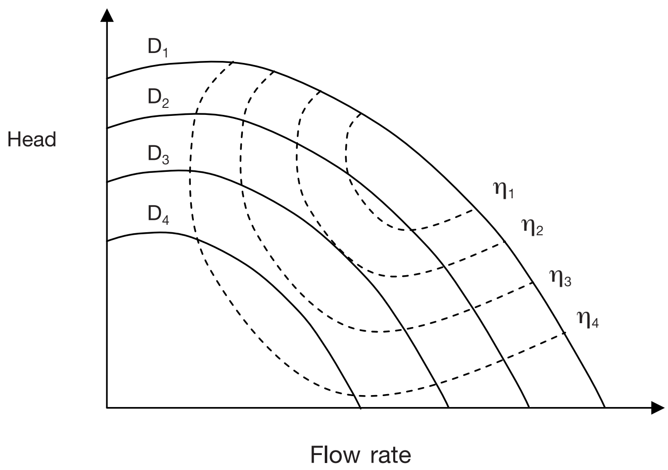

Fig. 4 Pump performance curves at a constant pump speed provided by pump manufacturers

As pointed out in Section 3, the performances curves of a theoretical pump are completely specified by 5 quantities: η_d, H_d, Q_d, H_m and Q_m. These quantities can be obtained from commercial pump performance curves. It can be demonstrated that the performance curves of the 5 commercial pumps listed in Table 1 can be modeled by Eqs. (12) and (13). Table 2 shows the 5 quantities for each of the commercial pumps. Head curves and efficiency curves corresponding to these parameters are plotted and compared with data points extracted from commercial pump curves for each pump in Figs. 5 and 6. It can be seen that agreement between theoretical pump curves and commercial pump curves is quite good.

Table 2 Maximum efficiency (η_d), design head (H_d), design flow rate (Q_d), maximum head (H_m) and the corresponding flow rate (Q_m) for each of the 5 commercial pumps

Pump Number	η_d (%)	H_d (m)	Q_d (m^3/h)	H_m (m)	Q_m (m^3/h)
1	79.34	46.42	289.4	63.89	71.80
2	81.78	67.64	165.3	74.50	72.80
3	79.63	147.8	245.2	185.8	41.25
4	59.31	47.12	18.72	58.45	2.300
5	50.65	18.74	11.87	23.29	1.269

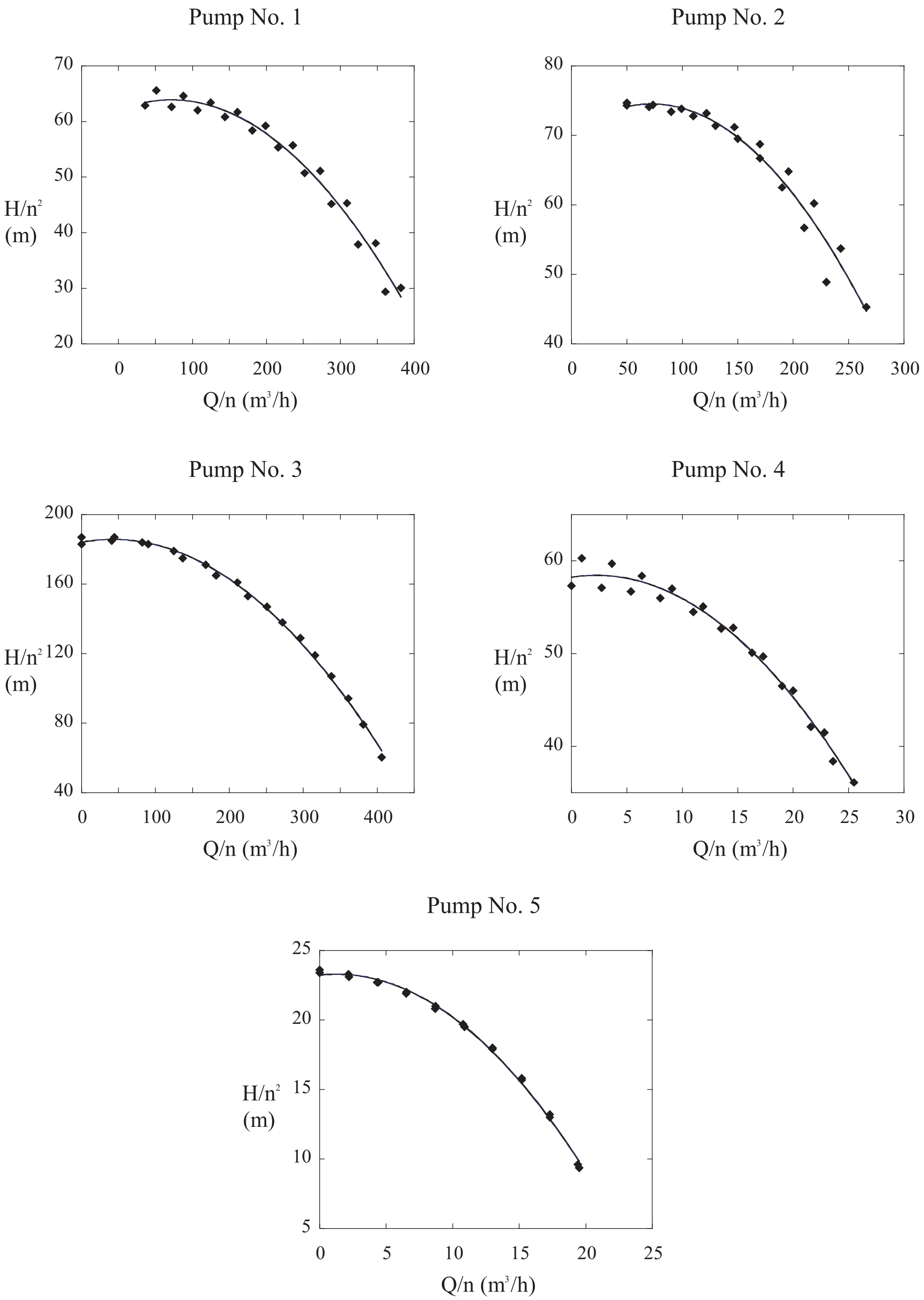

Fig. 5 Comparison between data points extracted from head curves of 5 pumps and head curves of corresponding theoretical pumps

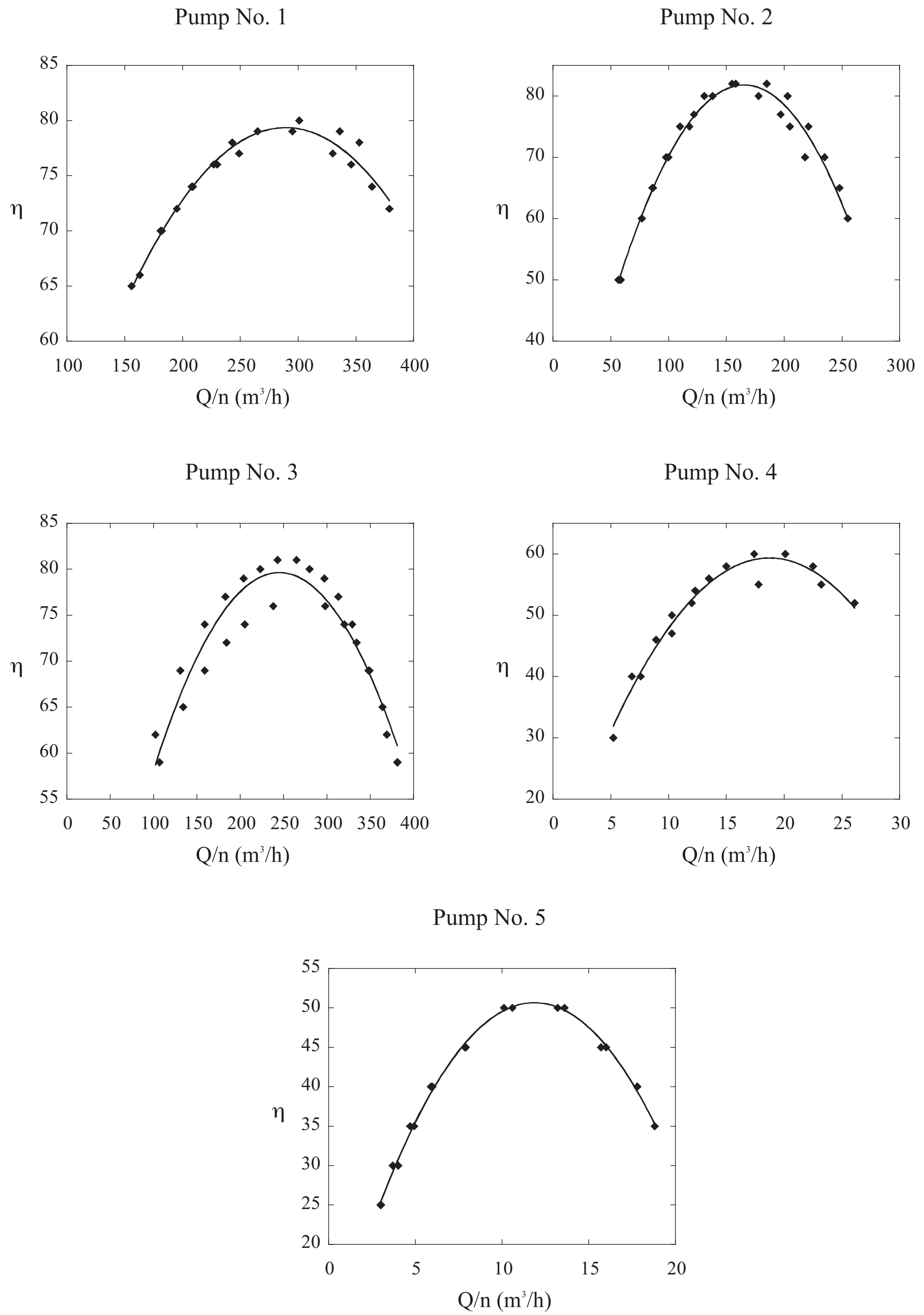

Fig. 6 Comparison between data points extracted from efficiency curves of 5 pumps and efficiency curves of corresponding theoretical pumps

5. Development of Formulas

It is assumed that a pump is originally designed to operate at the design speed (N_d) and the design operating point (Q_d, H_d). The design flow rate is assumed to be too high, and it is desired to reduce flow rate from Q_d to Q_r by operating the pump at a reduced speed (N_r). In order to find N_r, it is necessary to make an assumption about the system curve. It has been suggested that the system curve should be approximated by a parabola. This suggestion is adopted in this paper, and the system equation is assumed to be

$$H = KQ^2 + H_s \qquad \textbf{(21)}$$

where K depends on the amount of friction in the system, and H_S is the static head of the system. The static head is considered to be a free parameter, whereas the K can be determined from the fact that the design point (Q_D, H_D) lies on the system curve.

$$K = \frac{\left(H_d - H_s\right)}{Q_d^2} \qquad \textbf{(22)}$$

The dimensionless equation corresponding to Eq. (21) is

$$h = (1 - h_s)q^2 + h_s \qquad \textbf{(23)}$$

Since an operation point of the pump system is at an intersection between the pump head curve and the system curve, N_r can be found by solving the following equation:

$$\frac{h_m\left(q_0 n_r - q_r\right)\left(q_0 n_r - 2q_m n_r + q_r\right)}{\left(q_0 - q_m\right)^2} = \left(\ \ - h_s\right)q_r^2 + h_s \qquad \textbf{(24)}$$

where $n_r = N_r/N_d$. Once n_r is known, the pump power (p_r) at the reduced flow rate can be determined from Eq. (18). It can be seen from Eq. (24) that both n_r and p_r depend on only 3 parameters: h_s, q_m and h_m. It is interesting to note that, for the 5 commercial pumps, values of q_m range from 0.107 to 0.440, and values of h_m range from 1.101 to 1.376.

For given values of q_m and h_m, n_r and p_r depend only on h_S. According to the affinity laws, $n_r = q_r$ when $h_S = 0$. For non-zero values of h_S, n_r is a function of both q_r and h_S. Although many functional forms can be chosen for n_r, it is found that the functional relationship between $n_r - q_r$ and $1 - q_r$ is relatively simple. For instance, Fig. 5 shows plots of variations of $n_r - q_r$ with $1 - q_r$ for $q_m = 0.3$, $h_m = 1.4$ and 5 values of h_S. Inspection of Fig. 7 reveals that $n_r - q_r$ may be approximated as a quadratic function of $1 - q_r$ if h_S is held constant, and $n_r - q_r$ may be approximated as a quadratic function of h_S if q_r is held constant. Therefore, the expression of $n_r - q_r$ may be written as

$$n_r = q_r + \alpha_1 h_s\left(1 - q_r\right) + \alpha_2 h_s\left(1 - q_r\right)^2 + \alpha_3 h_s^2\left(1 - q_r\right)^2 \qquad \textbf{(25)}$$

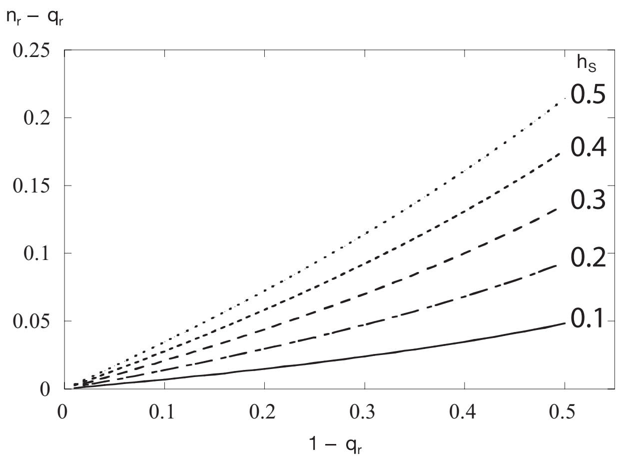

Fig. 7 Variations of $n_r - q_r$ with $1 - q_r$ for $q_m = 0.3$, $h_m = 1.4$ and 5 values of h_S

Values η_1, η_2 and η_3 are determined by curve fitting procedure, and they depend on both q_m and h_m. It can be seen that Eq. (1) is applicable only when $h_S = 0$. Values of η_1, η_2 and η_3 for different values of q_m and h_m are given in Tables 3, 4 and 5.

Table 3 Values of coefficient α_1 corresponding to selected values of q_m and h_m.

q_m \ h_m	1.1	1.2	1.3	1.4	1.5	1.6
0.0	0.879	0.801	0.736	0.682	0.634	0.592
0.1	0.869	0.786	0.716	0.659	0.608	0.566
0.2	0.857	0.766	0.692	0.632	0.580	0.536
0.3	0.841	0.742	0.662	0.598	0.545	0.501
0.4	0.821	0.711	0.626	0.559	0.504	0.459
0.5	0.794	0.669	0.580	0.509	0.454	0.410

Table 4 Values of coefficient α_2 corresponding to selected values of q_m and h_m.

q_m \ h_m	1.1	1.2	1.3	1.4	1.5	1.6
0.0	1.006	0.937	0.886	0.832	0.789	0.745
0.1	1.000	0.932	0.875	0.817	0.771	0.727
0.2	0.996	0.921	0.858	0.802	0.747	0.703
0.3	0.988	0.910	0.842	0.779	0.721	0.676
0.4	0.984	0.897	0.820	0.747	0.691	0.642
0.5	0.981	0.880	0.790	0.714	0.650	0.594

Table 5 Values of coefficient α_3 corresponding to selected values of q_m and h_m.

q_m \ h_m	1.1	1.2	1.3	1.4	1.5	1.6
0.0	1.228	1.078	0.976	0.876	0.794	0.717
0.1	1.201	1.053	0.935	0.825	0.742	0.667
0.2	1.178	1.008	0.876	0.767	0.667	0.600
0.3	1.135	0.951	0.808	0.683	0.583	0.516
0.4	1.092	0.867	0.708	0.567	0.476	0.408
0.5	1.019	0.748	0.558	0.425	0.332	0.257

The affinity laws also suggest that there is a relatively simple functional relationship between $\log(p_r)$ and $\log(q_r)$. Figure 8 shows plots of variations of $\log(p_r)$ with $\log(q_r)$ for $q_m = 0.3$, $h_m = 1.4$ and 5 values of h_S. Inspection of Fig. 8 reveals that $\log(p_r)$ may be approximated as a quadratic function of $\log(q_r)$ if h_s is held constant, and $\log(p_r)$ may be approximated as a quadratic function of h_S if $\log(q_r)$ is held constant. Therefore, the expression of $\log(p_r)$ may be written as

$$\log(p_r) \quad = \quad 3\log(q_r) - \beta_1 h_s \log(q_r) + \beta_2 h_s [\log(q_r)]^2 - \beta_3 h_s^2 [\log(q_r)]^2 \qquad \textbf{(26)}$$

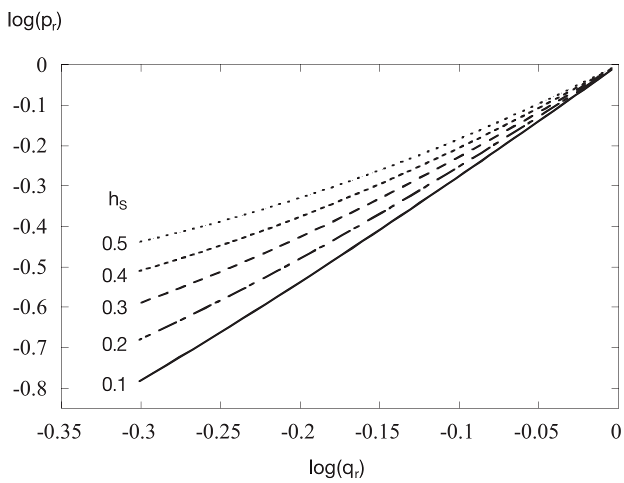

Fig. 8 Variations of $\log(p_r)$ with $\log(q_r)$ for $q_m = 0.3$, $h_m = 1.4$ and 5 values of h_S

Values β_1, β_2 and β_3 are determined by curve fitting procedure, and they depend on both q_m and h_m. It can be seen that Eq. (4) is applicable only when $h_S = 0$. Values of β_1, β_2 and β_3 for different values of q_m and h_m are given in Tables 6, 7 and 8.

Table 6 Values of coefficient β_1 corresponding to selected values of q_m and h_m.

h_m / q_m	1.1	1.2	1.3	1.4	1.5	1.6
0.0	1.954	1.954	1.954	1.949	1.944	1.935
0.1	1.954	1.954	1.952	1.947	1.935	1.923
0.2	1.953	1.952	1.948	1.939	1.924	1.904
0.3	1.953	1.949	1.942	1.926	1.902	1.872
0.4	1.952	1.944	1.929	1.902	1.863	1.813
0.5	1.949	1.934	1.901	1.849	1.780	1.696

Table 7 Values of coefficient β_2 corresponding to selected values of q_m and h_m.

h_m / q_m	1.1	1.2	1.3	1.4	1.5	1.6
0.0	7.812	7.711	7.723	7.792	7.902	8.030
0.1	7.795	7.718	7.761	7.874	8.015	8.169
0.2	7.778	7.741	7.833	7.991	8.165	8.347
0.3	7.776	7.793	7.952	8.159	8.397	8.563
0.4	7.772	7.888	8.136	8.407	8.636	8.804
0.5	7.817	8.077	8.428	8.729	8.913	8.940

Table 8 Values of coefficient β_3 corresponding to selected values of q_m and h_m.

h_m / q_m	1.1	1.2	1.3	1.4	1.5	1.6
0.0	8.982	8.910	8.894	8.860	8.796	8.660
0.1	8.966	8.896	8.867	8.823	8.694	8.473
0.2	8.942	8.885	8.841	8.735	8.491	8.112
0.3	8.933	8.867	8.780	8.539	8.094	7.407
0.4	8.885	8.814	8.600	8.092	7.212	5.950
0.5	8.858	8.683	8.100	6.919	5.073	2.517

With sufficient information about a variable-speed pumping system, the pump speed and the power input of the pump at a given flow rate can be determined by using the presented method. First, the static head, the design head, the design flow rate, the maximum head, and the corresponding flow rate of the pump are needed for the computation of q_r, h_s, h_m and q_m. Next, $\alpha_1 - \alpha_3$ and $\beta_1 - \beta_3$ are found by interpolation from Tables 3 – 8. Equations (25) and (26) are then used to find the values of n_r and p_r. Finally, the desired pump speed and the power input are computed from

$$N_r = n_r N_d \quad (27)$$

$$P_r = \frac{\gamma Q_d H_d n_r}{\eta_d} \quad (28)$$

Table 9 shows computational results for the 5 commercial pumps.

Table 9 Computation of pump speeds and power inputs for the 5 commercial pumps with $q_r = 0.8$ and $h_s = 0.4$.

Pump Number	1	2	3	4	5
h_m	1.376	1.101	1.257	1.240	1.242
q_m	0.248	0.440	0.168	0.123	0.107
α_1	1.936	0.809	0.731	0.753	0.755
α_2	0.805	0.982	0.890	0.906	0.907
α_3	0.755	1.060	0.950	0.994	1.000
β_1	1.936	1.951	1.951	1.953	1.953
β_2	8.028	7.792	7.777	7.745	7.739
β_3	8.682	8.873	8.866	8.880	8.883
n_r	0.858	0.873	0.866	0.868	0.868
p_r	0.632	0.631	0.631	0.631	0.631
N_r (rpm)	2544	2514	3087	3038	2517
P_r (kW)	29.16	23.51	78.26	2.557	0.755

6. Conclusion

The derivation of formulas for computing power input of a theoretical centrifugal pump operating at an arbitrary speed is presented. The head of the theoretical pump is a quadratic function of flow rate, whereas the efficiency is a cubic function. It is found that pump performance curves of 5 commercial pumps from 5 manufacturers resemble performance curves of theoretical pumps. Together with the assumption that the system curve is a parabola, these curves are used to determine the speed and the power input of a pump at a given flow rate. Curve fitting of computational results yield formulas of speed and power input as functions of flow rate, the static head, and 5 pump characteristics (maximum efficiency, design head, design flow rate, maximum head and the corresponding flow rate).

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