

## สัมประสิทธิ์การแผ่รังสีความร้อนในอุปกรณ์แลกเปลี่ยนความร้อนแบบท่อคู่ โดยที่ท่อในมีอุณหภูมิคงที่และท่อนอกหุ้มฉนวน

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### บทคัดย่อ

การวิเคราะห์การทำงานของอุปกรณ์แลกเปลี่ยนความร้อนแบบท่อคู่โดยพิจารณาเพียงการพาความร้อนแบบบังคับอาจให้ผลที่ได้คลาดเคลื่อนเนื่องจากการแผ่รังสีความร้อน โดยเฉพาะอย่างยิ่งในกรณีที่ก๊าซที่ไหลในอุปกรณ์มีอุณหภูมิสูง บทความนี้นำเสนอแนวทางการวิเคราะห์ที่พิจารณาการแผ่รังสีความร้อนเพื่อกำหนดค่าสัมประสิทธิ์การแผ่รังสีความร้อน ซึ่งควรใช้ร่วมกับค่าสัมประสิทธิ์การพาความร้อน เพื่อให้ผลการวิเคราะห์ที่มีความแม่นยำมากขึ้น โดยในกรณีนี้ท่อในของอุปกรณ์แลกเปลี่ยนความร้อนมีอุณหภูมิคงที่และท่อนอกหุ้มฉนวน ของไหลในอุปกรณ์คือ อากาศที่ไม่มีส่วนร่วมกับการแลกเปลี่ยนรังสีความร้อน พื้นผิวของท่อในและท่อนอกแบ่งเป็นชั้นๆ ซึ่งมีอุณหภูมิคงที่ ระบบสมการไม่เชิงเส้นที่ใช้ในการศึกษาประกอบด้วยสมการแลกเปลี่ยนรังสีความร้อนระหว่างพื้นผิวของชั้นประกอบเหล่านี้ และสมการสมดุลพลังงานของอากาศ ผลเฉลยที่ได้คือการกระจายอุณหภูมิภายในอุปกรณ์ ซึ่งใช้หาค่าสัมประสิทธิ์การแผ่รังสีความร้อนได้ ค่าสัมประสิทธิ์การถ่ายเทความร้อนเป็นฟังก์ชันของอุณหภูมิเฉลี่ยของก๊าซและอุณหภูมิของท่อใน และต้องอาศัยแพดเตอร์ปรับแก้ ซึ่งไม่ใช่ค่าคงที่ แต่จะเปลี่ยนแปลงตามเลขเรย์โนลด์และค่าการเปล่งรังสีของท่อในและท่อนอก

**คำสำคัญ :** อุปกรณ์แลกเปลี่ยนความร้อน / การแผ่รังสีความร้อน / การพาความร้อนแบบบังคับ

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## **Radiative Heat Transfer Coefficient in Double-Pipe Heat Exchanger with Isothermal Inner Pipe and Insulated Outer Pipe**

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### **Abstract**

Analysis of a double-pipe heat exchanger that considers only forced convection may yield inaccurate results due to the fact that there is also thermal radiation, especially when the gas flowing in the heat exchanger has a higher temperature. This article presents a method of analysis that takes into account thermal radiation in order to determine the radiation heat transfer coefficient, which should be used in conjunction with convective heat transfer coefficient in order to obtain more accurate results. An inner pipe of the heat exchanger is assumed to be at a constant temperature, whereas the outer pipe is insulated. The fluid is air that is transparent to radiation. The surfaces of the inner and outer pipes are divided into small elements that have uniform temperatures. The nonlinear system of equations of radiative exchange and energy balance equations is solved for temperature distributions, from which the radiative heat transfer coefficient is determined. The radiative heat transfer coefficient is expressed in terms of the average gas temperature and temperature of the inner pipe. It is found that a correction factor must be inserted into this expression. In addition, the correction factor is not a constant, but varies with Reynolds number and emissivities of the inner and outer pipes.

**Keywords :** Heat Exchanger / Thermal Radiation / Forced Convection

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## 1. Introduction

Although many textbooks in heat transfer assume that thermal radiation is negligible in presence of forced convection, a scale analysis by Kottke et al. [1] indicates that there are situations in which the magnitude of thermal radiation is either comparable to or greater than that of forced convection. An accurate analysis, therefore, requires that thermal radiation should be included. However, thermal radiation heat transfer is a complex phenomenon that is governed by integro-differential equations. Previous analyses of combined forced convection and thermal radiation are limited to simple flow geometries such as flow in circular tubes [2, 3], flow in noncircular tubes [4, 5], flow in a helical pipe [6], flow between parallel plates [7, 8], and flow in a square duct with a centered circular core [9].

Since flow in heat exchangers is mostly characterized by complicated geometries, an analysis of combined forced convection and thermal radiation in a heat exchanger is quite a challenge. A simplified analysis assumes that the overall heat transfer coefficient is the sum of convective and radiative heat transfer coefficients. Formulas for radiative coefficients are given in terms of temperatures of radiating surfaces and gas volumes [10 – 13]. The justification for these formulas is that the radiative transfer from non-isothermal surfaces can be approximated by that of isothermal surfaces and volumes with suitable representative temperatures. This paper investigates the validity of using such simple formulas to determine radiative heat transfer

coefficients in heat exchangers. The type of heat exchanger under consideration is double-pipe heat exchanger because its geometry is simpler than other types, and well-known correlation of convective heat transfer coefficient is available. The inner pipe is at a constant temperature, whereas the outer pipe is insulated. This heat exchanger may be considered as an evaporator with evaporating water flowing in the inner pipe and hot gas flowing in the outer pipe. The gas is assumed to be a non-participating medium in the radiative transfer, and tube surfaces are assumed to be gray and diffuse.

## 2. Heat Transfer in Double-pipe Heat Exchanger

Figure 1 shows the cross section of the double-pipe heat exchanger. Its length is  $L$ , its inside radius is  $r_1$ , and its outside radius is  $r_2$ . The inner pipe is at a uniform temperature of  $T_r$ . The inlet gas temperature is  $T_{f,0}$ . The outer pipe is insulated, which results in the non-uniform outer pipe temperature  $T_w(x)$ . With the specification of  $L$ ,  $r_1$ ,  $r_2$ ,  $T_r$ , and the Reynolds number, the variations of gas and outer pipe temperatures with position along the heat exchanger can be determined, which will yield results for the total heat transfer between the gas and the inner pipe and the overall heat transfer coefficient.

The correlation of Nusselt number for a turbulent flow in the annular space between two concentric cylindrical pipes is given by Gnielinski [14].

$$\text{Nu} = \frac{(f_{ann}/8)(\text{Re} - 1000)\text{Pr}}{1 + 12.7(f_{ann}/8)^{1/2}(\text{Pr}^{2/3} - 1)} \left[ 1 + \left( \frac{d_h}{L} \right)^{2/3} \right] F_{ann} K \quad (1)$$

where  $d_h = 2(r_o - r_i)$ , and  $K = 1$  for a gas being cooled. It should be noted that the length scale in Re and Nu is the hydraulic diameter ( $d_h$ ), and that gas properties are evaluated at the average gas temperature. According to Gnielinski [14], the expressions for  $f_{ann}$  and  $F_{ann}$  are

$$f_{ann} = (1.8 \log Re^* - 1.5)^{-2} \tag{2}$$

$$Re^* = Re \frac{[1 + (r_i/r_o)^2] \ln(r_i/r_o) + [1 - (r_i/r_o)^2]}{[1 - (r_i/r_o)^2] \ln(r_i/r_o)} \tag{3}$$

$$F_{ann} = 0.75 \left( \frac{r_i}{r_o} \right)^{-0.17} \tag{4}$$

Equation (4) is for the case of insulated outer pipe and convective heat transfer occurring between the inner pipe and the gas. However, even though the outer pipe is insulated in Fig. 1, there is still convective heat transfer between the outer pipe and the gas since the radiative heat transfer results in the outer pipe having higher temperature than the gas. According to Gnielinski [15], there are no experimental data for the case of convective heat transfer from both the inner and outer pipes to the gas. Therefore, Eq. (4) is assumed to be applicable in this case. The above equations are used to obtain convective heat transfer coefficient between the gas and the concentric cylinders, which is assumed to be independent of position along the cylinders.

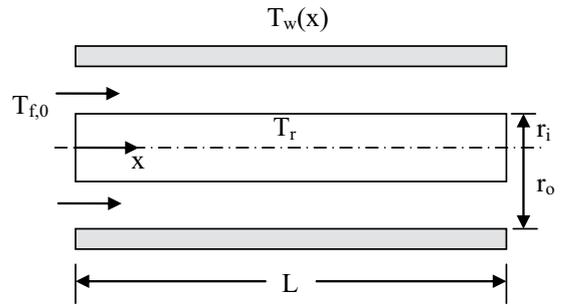


Fig. 1 Cross section of double-pipe heat exchanger.

In order to determine thermal radiation heat transfer in the heat exchanger, tube surfaces are divided into  $n$  isothermal elements. The length of each element is  $\Delta x = L/n$ . For the element  $i$  shown in Fig. 2, the inlet and exit gas temperatures are, respectively,  $T_{f,i-1}$  and  $T_{f,i}$ , whereas the temperature of the outer pipe is uniformly  $T_{w,i}$ . The energy balance of this element is

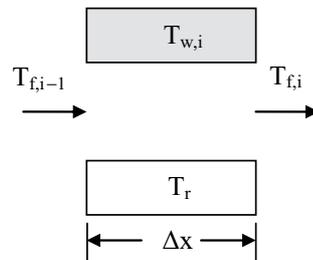


Fig. 2 Element  $i$  of the double-pipe heat exchanger.

$$C(T_{f,i} - T_{f,i-1}) = hP_i \Delta x \left[ T_r - \frac{1}{2}(T_{f,i-1} + T_{f,i}) \right] + hP_o \Delta x \left[ T_{w,i} - \frac{1}{2}(T_{f,i-1} + T_{f,i}) \right] \tag{5}$$

where  $C$  is the product of mass flow rate and specific heat capacity of the gas,  $P_i = 2\pi r_i L/n$  and  $P_o = 2\pi r_o L/n$ .

The division of the heat exchanger into  $n$  elements results in the creation of  $2n + 2$  radiating

surfaces. Let surfaces 1 to  $n$  denote the  $n$  surfaces of the inner pipe,  $n + 1$  to  $2n$  denote the  $n$  surfaces of the outer pipe,  $2n + 1$  and  $2n + 2$  denote the imaginary annular surfaces at the left and right ends with surface temperature  $T_L$  and  $T_R$ , respectively.

Surfaces 1 – 2*n* are assumed to be gray surfaces. The emissivity of surfaces 1 – *n* is  $\epsilon_1$ , and the emissivity of surfaces *n* + 1 to 2*n* is  $\epsilon_2$ . Surfaces 2*n* + 1 and 2*n* + 2

are black surfaces. Equations of radiative transfer among the 2*n* + 2 surfaces can be written as [16]

$$\begin{aligned} \frac{q_j}{\epsilon_1} + \frac{h}{\epsilon_1} \left[ \frac{1}{2} (T_{f,j-1} + T_{f,j}) - T_r \right] - \sum_{k=1}^n \frac{h(1-\epsilon_2)}{\epsilon_2} \left[ \frac{1}{2} (T_{f,k-1} + T_{f,k}) - T_{w,k} \right] F_{j,n+k} \\ = \sigma T_r^4 - \sigma \sum_{k=1}^n F_{j,n+k} T_{w,k}^4 - \sigma F_{j,2n+1} T_L^4 - \sigma F_{j,2n+2} T_R^4 \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{h}{\epsilon_2} \left[ \frac{1}{2} (T_{f,j-1} + T_{f,j}) - T_{w,j} \right] - \sum_{k=1}^n \frac{h(1-\epsilon_1)}{\epsilon_1} \left[ \frac{1}{2} (T_{f,k-1} + T_{f,k}) - T_r \right] F_{n+j,k} \\ - \sum_{k=1}^n \frac{h(1-\epsilon_2)}{\epsilon_2} \left[ \frac{1}{2} (T_{f,k-1} + T_{f,k}) - T_{w,k} \right] F_{n+j,n+k} \\ = \sigma T_{w,j}^4 - \sigma \sum_{k=1}^n F_{n+j,k} T_r^4 - \sigma \sum_{k=1}^n F_{n+j,n+k} T_{w,k}^4 - \sigma F_{n+j,2n+1} T_L^4 - \sigma F_{n+j,2n+2} T_R^4 \end{aligned} \quad (7)$$

for  $j = 1, 2, \dots, n$ , and

$$\begin{aligned} - \sum_{k=1}^n \frac{h(1-\epsilon_1)}{\epsilon_1} \left[ \frac{1}{2} (T_{f,k-1} + T_{f,k}) - T_r \right] F_{2n+1,k} - \sum_{k=1}^n \frac{h(1-\epsilon_2)}{\epsilon_2} \left[ \frac{1}{2} (T_{f,k-1} + T_{f,k}) - T_{w,k} \right] F_{2n+1,n+k} \\ = \sigma T_{w,j}^4 - \sigma \sum_{k=1}^n F_{2n+1,k} T_r^4 - \sigma \sum_{k=1}^n F_{2n+1,n+k} T_{w,k}^4 - \sigma F_{2n+1,2n+2} T_R^4 \end{aligned} \quad (8)$$

$$\begin{aligned} - \sum_{k=1}^n \frac{h(1-\epsilon_1)}{\epsilon_1} \left[ \frac{1}{2} (T_{f,k-1} + T_{f,k}) - T_r \right] F_{2n+2,k} - \sum_{k=1}^n \frac{h(1-\epsilon_2)}{\epsilon_2} \left[ \frac{1}{2} (T_{f,k-1} + T_{f,k}) - T_{w,k} \right] F_{2n+2,n+k} \\ = \sigma T_{w,j}^4 - \sigma \sum_{k=1}^n F_{2n+2,k} T_r^4 - \sigma \sum_{k=1}^n F_{2n+2,n+k} T_{w,k}^4 - \sigma F_{2n+2,2n+1} T_L^4 \end{aligned} \quad (9)$$

where  $F_{j,k}$  is view factor from surface *j* to surface *k*. Equations (6) – (9) represent a system of 2*n* + 2 nonlinear equations with 2*n* + 2 unknowns ( $q_1, q_2, \dots, q_n, T_{w,1}, T_{w,2}, \dots, T_{w,n}, T_L$ , and  $T_R$ ). It must be solved by iteration, starting with initial values of  $T_{f,1}, T_{f,2}, \dots, T_{f,n}, T_{w,1}, T_{w,2}, \dots$ , and  $T_{w,n}$ . For a convective flow without thermal radiation, the temperatures of the gas and the outer pipe are

$$T_{f,i} = T_r + (T_{f,0} - T_r) \exp \left[ - \frac{2\pi r_i h_i \Delta x}{C} \right] \quad (10)$$

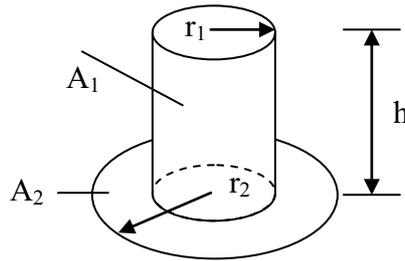
$$T_{w,i} = \frac{1}{2} (T_{f,i-1} + T_{f,i}) \quad (11)$$

They may be used as starting values in the iterative process to obtain the solution of Eqs. (6) – (9). Once  $T_{w,1}, T_{w,2}, \dots$ , and  $T_{w,n}$  are known,  $T_{f,1}, T_{f,2}, \dots$ , and  $T_{f,n}$ , are determined from Eq. (5).

### 3. View Factors

The solution of Eqs. (6) – (9) requires values of view factors. Sparrow et al. [17] give the following formula for view factor between the outer surface

( $A_1$ ) of a cylinder of radius  $r_1$  and length  $h$  and the annular surface ( $A_2$ ) between a circle of radius  $r_1$  at one end of the cylinder and a concentric circle of radius  $r_2$  as shown in Fig. 3.

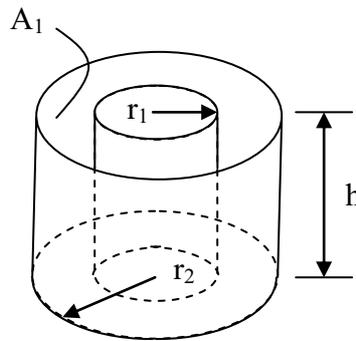


**Fig. 3** The outer surface ( $A_1$ ) of a cylinder that exchanges thermal radiation with the annular surface ( $A_2$ ) between the circle at one end of the cylinder and a concentric larger circle.

$$F_{1,2} = \frac{B}{8RH} + \frac{1}{2\pi} \left\{ \cos^{-1} \left( \frac{A}{B} \right) - \frac{1}{2H} \left[ \frac{(A+2)^2}{R^2} - 4 \right]^{1/2} \cos^{-1} \left( \frac{AR}{B} \right) - \frac{A}{2RH} \sin^{-1} R \right\} \quad (12)$$

where  $R = r_1/r_2$ ,  $H = h/r_2$ ,  $A = H^2 + R^2 - 1$ , and  $B = H^2 - R^2 + 1$ . Sparrow et al. [17] also give the following formula for view factor between the inner

surface ( $A_1$ ) of the outer cylinder of radius  $r_2$  and length  $h$  and itself in presence of the inner cylinder of radius  $r_1$  and the same length as shown in Fig. 4.



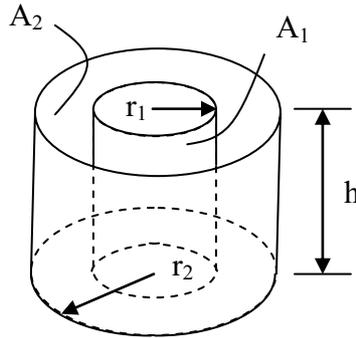
**Fig. 4** The inner surface ( $A_1$ ) of the outer cylinder that exchanges thermal radiation with itself in presence of the coaxial inner cylinder of the same length.

$$F_{1,1} = \frac{1}{\pi R_2} \left\{ \pi(R_2 - R_1) + \cos^{-1} \left( \frac{R_1}{R_2} \right) - (1 + 4R_2^2)^{1/2} \tan^{-1} \left[ \frac{[(1 + 4R_2^2)(R_2^2 - R_1^2)]^{1/2}}{R_1} \right] + 2R_1 \tan^{-1} \left[ 2(R_2^2 - R_1^2)^{1/2} \right] \right\} \quad (13)$$

where  $R_1 = r_1/h$ , and  $R_2 = r_2/h$ .

Brockmann [18] gives the following formula for view factor between the inner surface ( $A_1$ ) of

the outer cylinder of radius  $r_2$  and length  $h$  and the outer surface ( $A_2$ ) of the coaxial inner cylinder of radius  $r_1$  and the same length as shown in Fig. 5.

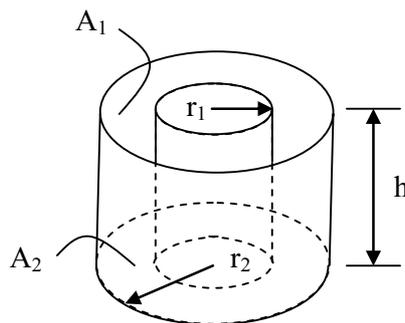


**Fig. 5** The inner surface ( $A_2$ ) of the outer cylinder that exchanges thermal radiation with the outer surface ( $A_1$ ) of the coaxial inner cylinder of the same length.

$$F_{1,2} = \frac{1}{\pi R_1} \left\{ \frac{1}{2} (R_2^2 - R_1^2 - 1) \cos^{-1} \left( \frac{R_1}{R_2} \right) + \pi R_1 - \frac{\pi}{2} AB - 2R_1 \tan^{-1} (R_2^2 - R_1^2)^{1/2} + \left[ (1 + A^2)(1 + B^2) \right]^{1/2} \tan^{-1} \left[ \frac{(1 + A^2)B}{(1 + B^2)A} \right]^{1/2} \right\} \quad (14)$$

where  $R_1 = r_1/h$ ,  $R_2 = r_2/h$ ,  $A = R_2 + R_1$ , and  $B = R_2 - R_1$ . Brockmann [18] also gives the following formula for view factor between the inner surface ( $A_1$ ) of the outer cylinder of radius  $r_2$  and length  $h$

and the annular surface ( $A_2$ ) enclosing the space between the outer cylinder and the coaxial inner cylinder of radius  $r_1$  as shown in Fig. 6.



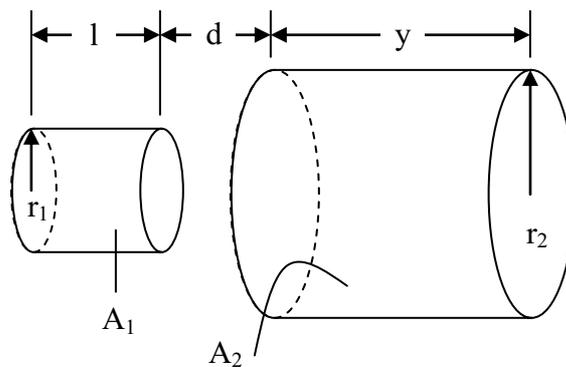
**Fig. 6** The inner surface ( $A_1$ ) of the outer cylinder that exchanges thermal radiation with the annular surface ( $A_2$ ) enclosing the space between the outer cylinder and the coaxial inner cylinder.

$$F_{1,2} = \frac{1}{\pi} \left\{ R \left( \tan^{-1} \frac{X}{H} - \tan^{-1} \frac{2X}{H} \right) + \frac{H}{4} \left[ \sin^{-1} (2R^2 - 1) - \sin^{-1} R \right] + \frac{X^2}{4H} \left( \frac{\pi}{2} + \sin^{-1} R \right) - \frac{\left[ (1 + R^2 + H^2)^2 - 4R^2 \right]^{1/2}}{4H} \left( \frac{\pi}{2} + \sin^{-1} Y \right) + \frac{(4 + H^2)^{1/2}}{4} \left[ \frac{\pi}{2} + \sin^{-1} \left( 1 - \frac{2R^2 H^2}{4X^2 + H^2} \right) \right] \right\} \quad (15)$$

where  $H = h/r_2$ ,  $X = (1 - R^2)^{1/2}$ ,  $R = r_1/r_2$ , and  $Y = R(1 - R^2 - H^2) / (1 - R^2 + H^2)$

Rea [19] gives the following formula for view factor between the outer surface ( $A_1$ ) of the inner

cylinder of radius  $r_1$  and length  $l$  and the inner surface ( $A_2$ ) of the coaxial outer cylinder of radius  $r_2$  and length  $y$  located at a distance  $d$  from the inner cylinder as shown in Fig. 7.



**Fig. 7** The inner surface ( $A_2$ ) of the outer cylinder that exchanges thermal radiation with the outer surface ( $A_1$ ) of the coaxial inner cylinder located away from the outer cylinder.

$$F_{1,2} = \frac{L + D}{L} f_{L+D} + \frac{Y + D}{L} f_{Y+D} - \frac{D}{L} f_D - \frac{L + D + Y}{L} f_{L+D+Y} \quad (16)$$

where

$$f_{\xi} = \frac{B_{\xi}}{8R\xi} + \frac{1}{2\pi} \left\{ \cos^{-1} \left( \frac{A_{\xi}}{B_{\xi}} \right) - \frac{A_{\xi}}{2\xi R} \sin^{-1} R - \frac{1}{2\xi} \left[ \frac{(A_{\xi} + 2)^2}{R^2} - 4 \right]^{1/2} \cos^{-1} \left( \frac{A_{\xi} R}{B_{\xi}} \right) \right\} \quad (17)$$

and  $D = d/r_2$ ,  $Y = y/r_2$ ,  $L = l/r_2$ ,  $R = r_1/r_2$ ,  $A_{\xi} = \xi^2 + R^2 - 1$ ,  $B_{\xi} = \xi^2 - R^2 + 1$ .

### 4. Results and Discussion

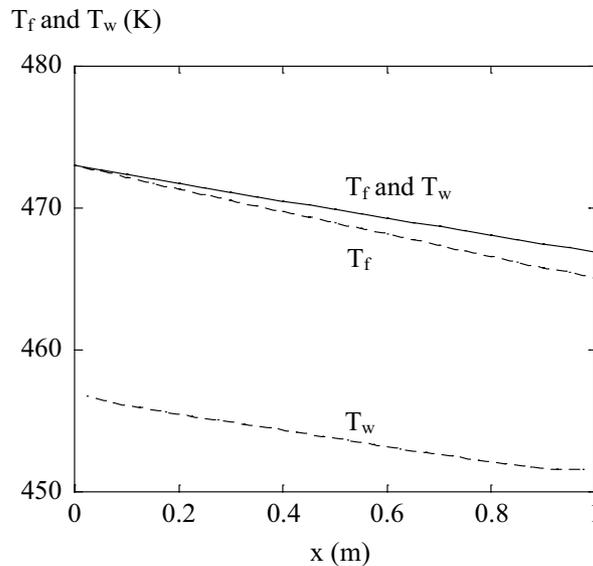
Simulation was performed for the base case, in which  $r_1 = 0.1$  m,  $r_2 = 0.2$  m,  $L = 1.0$  m,  $T_{f0} = 473$  K,  $T_r = 373$  K,  $Re = 5000$ , and  $\varepsilon_1 = \varepsilon_2 = \varepsilon = 0.2$ . It was found that results of simulation were not noticeably when the number of elements was increased from 10

to 20. Therefore, results to be shown in this section were obtained using 20 elements. Solution of Eqs. (6) – (9) yields the temperature distributions of the gas and the outer pipe along the heat exchanger as shown in Fig. 8. Also shown in Fig. 8 for comparison are the corresponding temperature distributions

obtained from Eqs. (10) and (11) when thermal radiation is ignored. It can be seen that the temperature distributions of the gas the outer pipe are identical when thermal radiation is ignored because the outer pipe is insulated. Combined forced convection and thermal radiation results in lower temperature profile for the gas and even lower temperature distribution for the outer pipe because more heat is transferred from the gas to the inner and outer pipes. Although the difference in temperature profiles of the gas appears to be small, the difference

in heat transfer is large. The heat transfer between the gas and the inner pipe is 375.5 W if thermal radiation is ignored, but it is 488.9 W if thermal radiation is included in the simulation. Higher heat transfer for the combined forced convection and thermal radiation yields a larger value of the overall heat transfer coefficient, which is determined as follows.

$$U = \frac{\dot{Q}}{A\Delta T_{lm}} \tag{18}$$



**Fig. 8** Profiles of gas and outer pipe temperatures along the heat exchanger for the base case when there is only forced convection (solid line) and when there is combined forced convection and thermal radiation (dashed lines).

where  $A$  is the area of the inner pipe, and the expression of the log-mean temperature difference is

$$\Delta T_{lm} = \frac{T_{f,0} - T_{f,n}}{\ln\left[\frac{T_{f,0} - T_r}{T_{f,n} - T_r}\right]} \tag{19}$$

The convective heat transfer coefficient for the base case is  $h = 6.165 \text{ W/m}^2\cdot\text{K}$ . The overall heat transfer

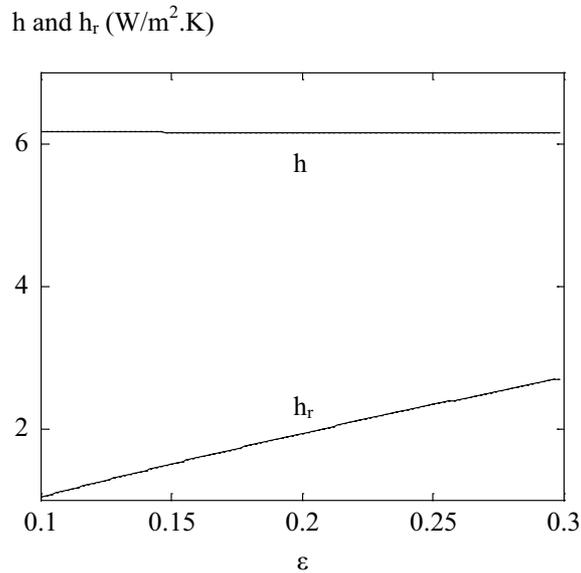
coefficient is found to be  $U = 8.106 \text{ W/m}^2\cdot\text{K}$ . The radiative heat transfer coefficient is defined as

$$h_r = U - h \tag{20}$$

For the base case,  $h_r = 1.941 \text{ W/m}^2\cdot\text{K}$  or 31.5% of the convective heat transfer coefficient.

In order to see how the radiative heat transfer coefficient is affected by emissivity, simulation was performed for a range of  $\varepsilon$  between 0.1 and 0.3. Results are shown in Fig. 9. It can be seen that increasing emissivity leads to increasing radiation exchange between the inner and outer pipe and lower temperatures of the outer pipe and the gas,

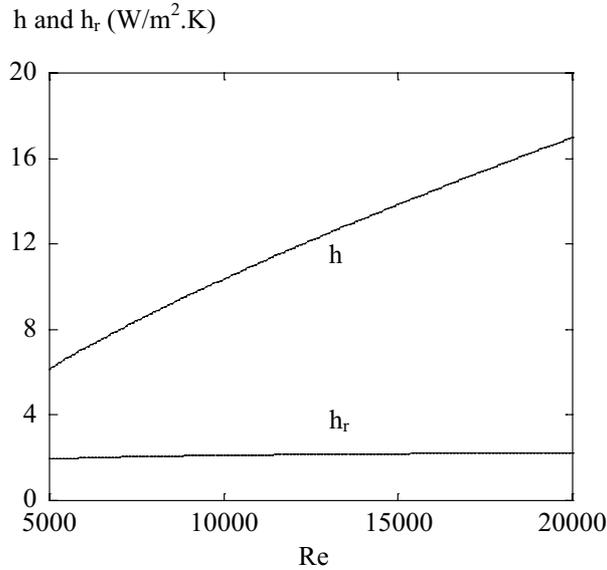
which results in a larger value of  $h_r$ . Increased emissivity affects  $h$  indirectly through changes in thermal properties of the gas as the temperature profile of the gas changes. Figure 9 shows that the effect of emissivity on  $h$  is very small, but the effect on  $h_r$  is significant.



**Fig. 9** Variations of convective and radiative heat transfer coefficients with emissivity of the inner and outer pipes.

Next, the effect of Reynolds number was investigated by varying  $Re$  from 5000 to 20000. It can be seen from Fig. 10 that increasing  $Re$  results in higher values of  $h$  in accordance with Eq. (1). It also enhances thermal radiation by causing the

difference between the gas temperature and the inner pipe temperature to increase, resulting in increasing  $h_r$ . Figure 10 shows that increasing Reynolds number has a stronger effect on convective heat transfer than radiative heat transfer.

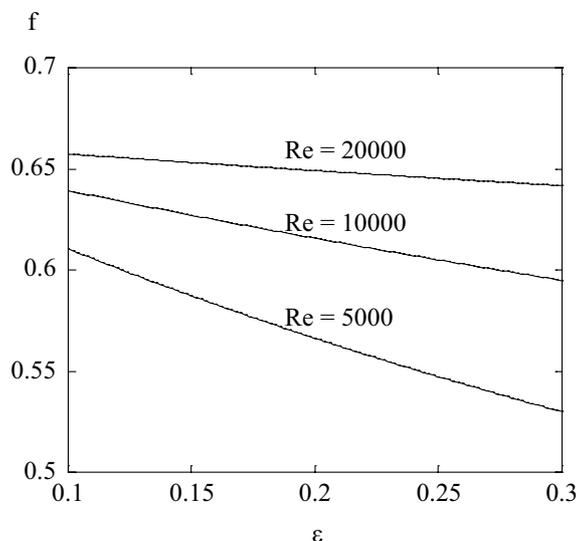


**Fig. 10** Variations of convective and radiative heat transfer coefficients with Reynolds number.

Instead of having to determine  $h_r$  from the solution of Eqs. (6) – (9), it would be convenient to have a formula for  $h_r$ . Such a formula may be written as

$$h_r = f \varepsilon \sigma (T_{f,n/2} + T_r) (T_{f,n/2}^2 + T_r^2) \quad (21)$$

where  $T_{f,n/2}$  is the gas temperature at the center of the heat exchanger, and  $f$  is the correction factor. A plot of the variation of  $f$  with  $\varepsilon$  for three values of  $Re$  in Fig. 11 shows that the correction factor is significantly smaller than 1. It decreases with increasing emissivity, but increases with Reynolds number.



**Fig. 11** Variations of correction factor with emissivity for three values of Reynolds number.

## 5. Conclusion

This article presents a method of analyzing a double-pipe heat exchanger in which the inner pipe is isothermal and the outer pipe is insulated. This method incorporates both forced convection and thermal radiation. The fluid is a gas that is transparent to radiation. The surfaces of the inner and outer pipes are divided into small elements that have uniform temperatures. The nonlinear system of equations of radiative exchange, along with energy balance equations of the gas, is solved for temperature distributions of the gas and the outer pipe. It is shown that the analysis using the assumption that there is only forced convection yields incorrect results. However, such an analysis may be improved by using the sum of convective and radiative heat transfer coefficient as the overall heat transfer coefficient. The radiative heat transfer coefficient is expressed in terms of average gas temperature, inner pipe temperature, and a correction factor that varies with Reynolds number and the emissivities of the inner and outer pipes.

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