การลดอันดับแบบจำลองของระบบพลวัตเชิงเส้นไม่เปลี่ยนแปลงตามเวลา ด้วยการค้นหาแบบนกกาเหว่า

สาโรจน์ หลั่งน้ำทิพย์[1] และ เดชา พวงดาวเรือง[2*]
มหาวิทยาลัยเอเชียอาคเนย์ แขวงหนองค้างพลู เขตหนองแขม กรุงเทพฯ 10160

บทคัดย่อ

บทความนี้นำเสนอการลดอันดับแบบจำลองของระบบพลวัตเชิงเส้นไม่เปลี่ยนแปลงตามเวลา (Linear Time-invariant (LTI) Dynamic Systems) ด้วยการค้นหาแบบนกกาเหว่า (Cuckoo Search: CS) ซึ่งเป็นเทคนิคการหาค่าเหมาะที่สุดเชิงอภิศึกษาสำนึกแบบอิงประชากรที่ทรงประสิทธิภาพ ระเบียบวิธีการลดอันดับแบบจำลองที่นำเสนอสามารถพิจารณาเป็นการหาค่าเหมาะที่สุดเชิงอภิศึกษาสำนึก ระบบอันดับสูงจำนวนสองระบบ ได้แก่ ระบบอันดับ-4 และระบบอันดับ-8 ซึ่งได้รับการคัดสรรมาจากงานวิจัยที่เกี่ยวข้อง และได้ถูกนำมาใช้เพื่อลดอันดับแบบจำลองเป็นระบบอันดับ-2 จากนั้นนำผลที่ได้จาก CS ไปเปรียบเทียบกับผลที่ได้จากขั้นตอนวิธีเชิงพันธุกรรม (Genetic Algorithm : GA) และการหาค่าเหมาะที่สุดแบบฝูงอนุภาค (Particle Swarm Optimization : PSO) ซึ่งเป็นเทคนิคการหาค่าเหมาะที่สุดเชิงอภิศึกษาสำนึกแบบอิงประชากรที่ได้รับความนิยมอย่างแพร่หลาย จากผลการจำลองสถานการณ์พบว่า แบบจำลองอันดับ-2 ที่ได้จาก CS สามารถรักษาพฤติกรรมทางพลวัตของระบบอันดับ-4 และระบบอันดับ-8 ไว้ได้อย่างถูกต้องกว่า GA และ PSO ผลตอบสนองในโดเมนเวลาและผลตอบสนองในโดเมนความถี่ของระบบที่ได้รับการลดอันดับแสดงความถูกต้องและความมีประสิทธิภาพของระเบียบวิธีที่นำเสนอ

คำสำคัญ : การลดอันดับแบบจำลอง / ระบบพลวัตเชิงเส้นไม่เปลี่ยนแปลงตามเวลา / การค้นหาแบบนกกาเหว่า / การหาค่าเหมาะที่สุดเชิงอภิศึกษาสำนึก

* *Corresponding Author : deachap@sau.ac.th*

1 *ผู้ช่วยศาสตราจารย์ สาขาวิชาวิศวกรรมไฟฟ้า คณะวิศวกรรมศาสตร์*

2 *รองศาสตราจารย์ สาขาวิชาวิศวกรรมไฟฟ้า บัณฑิตวิทยาลัย*

Model Order Reduction of Linear Time-Invariant Dynamic Systems via Cuckoo Search

Saroj Hlungnamthip[1] and Deacha Puangdownreong[2*]

Southeast Asia University, Nongkhangplu, Nongkhaem, Bangkok, 10160

Abstract

This article proposes a model order reduction method for linear time-invariant (LTI) dynamic systems via cuckoo search (CS), which is one of the most powerful population-based metaheuristic optimization search techniques. The proposed order reduction method can be classified as the metaheuristic optimization. In this work, two selected cases of higher-order systems from the literature, i.e., fourth-order and eighth-order systems, were reduced to the second-order models. Genetic algorithm (GA) and particle swarm optimization (PSO), two well-known population-based metaheuristic optimization search techniques, were also applied for performance comparison. It was found that the reduced second-order models obtained via the CS retain the system dynamic behavior of the original higher-order systems superior to those obtained via GA and PSO. The time-domain and frequency-domain responses of the reduced order models show the accuracy and efficiency of the proposed method.

Keywords : Model Order Reduction / Linear Time-Invariant Dynamic System / Cuckoo Search / Metaheuristic Optimization

* *Corresponding Author : deachap@sau.ac.th*

1 *Assistant Professor, Department of Electrical Engineering, Faculty of Engineering.*

2 *Associate Professor, Department of Electrical Engineering, Graduate School.*

1. Introduction

In system modeling process, several physical systems are interpreted and translated into mathematical model via higher-order differential equations. This leads to the higher-order model representation. The analysis and design of high-order models are complicated and tedious works. It becomes more complex when the order of the system increases. The approximation of higher-order systems by lower-order models retaining the dynamic behavior of the original system is one of the most important problems in control system theory. The use of a reduced-order model makes it easy to analysis, simulation and design. By literatures, numerous methods were proposed for order reduction of linear time-invariant (LTI) dynamic systems. For examples, aggregation method [1], Pade approximation [2], Routh stability technique [3], moment matching technique [4], L_∞ optimization technique [5] and Routh approximation [6] have been consecutively proposed for model order reduction. Although there are several available methods, no method provides the best results for all systems. Model order reduction can be classified into the class of optimization problem. Recently, the metaheuristic optimization search techniques (called shortly "metaheuristics") have been widely applied to solve optimization problems [7]. Two popular population-based metaheuristic optimization techniques are genetic algorithm (GA) [8] and particle swarm optimization (PSO) [9]. For GA, it has been widely applied to many real-world applications, for example, mixed model assembly line balancing [10] and model parameter identification [11]. For PSO, it has been also widely applied to many real-world applications, for example, control synthesis [12] and energy forecasting [13]. From literature reviews, many metaheuristics have been applied to model order reduction, for examples, order reduction by GA [14], PSO [15] and PSO associated with the bacterial foraging optimization (BFO) [16].

From literatures, the cuckoo search (CS) firstly proposed by Yang and Deb [17] is one of the most powerful population-based nature-inspired metaheuristic opt mization techniques. The CS algorithm is based on the obligate brood parasitic behaviour of some cuckoo species in combination with the Lévy flight behaviour of some birds and fruit flies. The CS has been successfully applied to several real-world engineering problems such as spring design optimization [18], welded beam design [18, 19], multiple disc brake [19] and control system design [20]. Moreover, the global convergent property of the CS algorithms has been proved and reported [21]. In this article, the CS is applied to model order reduction of two cases of higher-order LTI dynamic systems conducted from literatures. The reduced-order models obtained by CS will be compared with those obtained by GA and PSO for accuracy and efficiency comparison.

2. Model Order Reduction Problem

Let the n^{th} higher-order model be the s-domain transfer functions of the original higher-order system as $G(s)$ stated in (1), and let the r^{th} reduced-order model ($r < n$) be the s-domain transfer functions of the reduced-order system as $R(s)$ expressed in (2), respectively. For LTI dynamic systems, parameters d_i and e_j in (1) and a_i and b_j in (2) are defined as scalar constants. The objective is to find the r^{th} reduced-order model $R(s)$ such that it can retain the important properties of $G(s)$ for the same types of inputs.

$$G(s)=\frac{\sum_{i=0}^{n-1} d_i s^i}{\sum_{j=0}^{n} e_j s^j} = \frac{d_{n-1}s^{n-1}+d_{n-2}s^{n-2}+\cdots+d_1 s+d_0}{e_n s^n+e_{n-1}s^{n-1}+\cdots+e_1 s+e_0} \quad (1)$$

$$R(s)=\frac{\sum_{i=0}^{r-1} a_i s^i}{\sum_{j=0}^{r} b_j s^j} = \frac{a_{r-1}s^{r-1}+a_{r-2}s^{r-2}+\cdots+a_1 s+a_0}{b_r s^r+b_{r-1}s^{r-1}+\cdots+b_1 s+b_0} \quad (2)$$

Model order reduction can be classified as an optimization problem represented by the block diagram as shown in Figure 1, where y(t) is the output of the original higher-order system and $y^*(t)$ is the output of the reduced-order model. The integral-squared error of difference between $y(t)$ and $y^*(t)$ expressed in (3) is formulated as the objective function J. With the same input $x(t)$, J will be fed back to the metaheuristic tuning block to be minimized in order to find the appropriate parameters of the reduced-order model satisfying to their corresponding boundaries set as inequality constraints stated in (3), where $a_{i\_\min}$ and $a_{i\_\max}$ are lower and upper bounds of the numerator parameter a_i, while $b_{i\_\min}$ and $b_{i\_\max}$ are lower and upper bounds of the denominator parameter b_i, respectively. Once J is minimized, the appropriate parameters of the reduced-order model are successfully obtained.

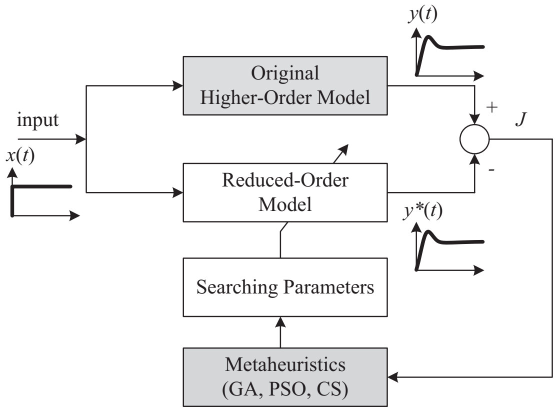


Figure 1 Model order reduction optimization problem

$$\begin{aligned} &\textbf{Minimize} && J=\int_0^{\tau}[y(t)-y^*(t)]^2\,dt \\ &\textit{subject to} && a_{i\_\min}\le a_i\le a_{i\_\max} \\ &&& b_{i\_\min}\le b_i\le b_{i\_\max} \end{aligned} \quad (3)$$

3. Cuckoo Search Algorithm

Originally, Yang and Deb proposed the cuckoo search (CS) in 2009 [17] as one of the most powerful population-based nature-inspired metaheuristic optimization techniques. In CS algorithm, each cuckoo lays one egg at a time, and dumps it in a randomly chosen nest. The best nests with high quality of eggs (solutions) will carry over to the next search iteration (generation). The number of available host nests is fixed, and a host can discover an alien egg with a probability $p_a \in [0, 1]$. The CS algorithms can be represented by the flow diagram as visualized in Figure 2.

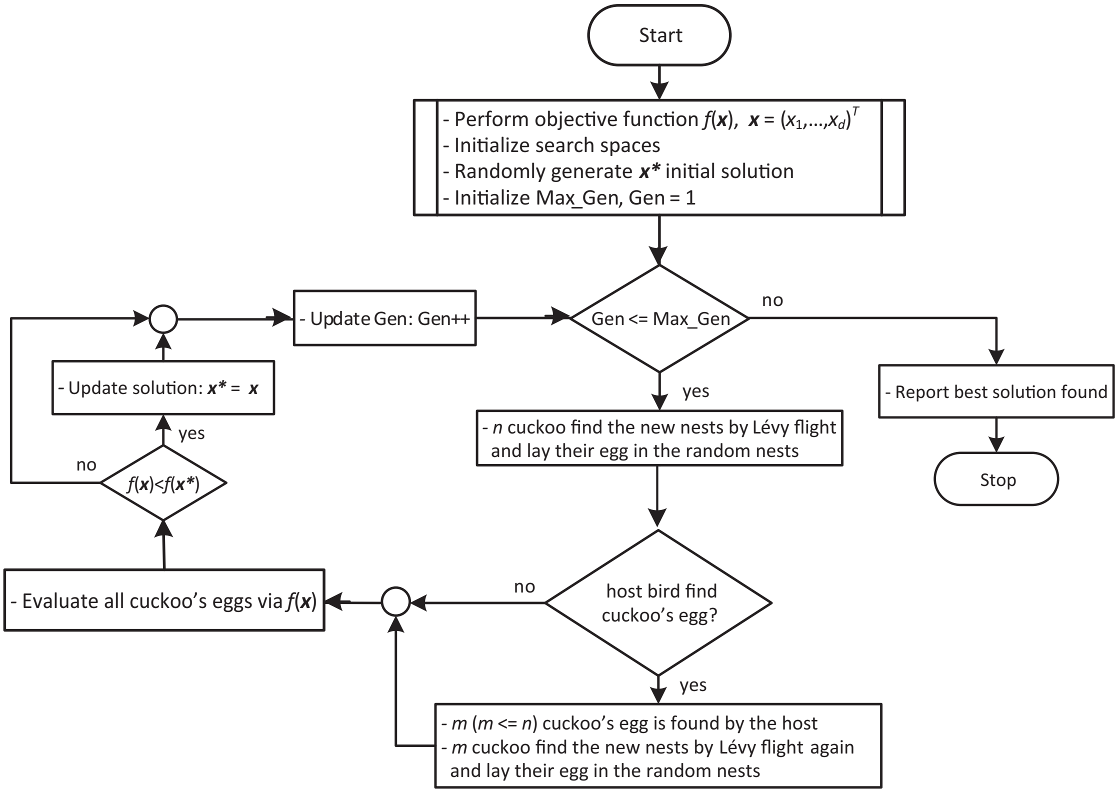


Figure 2 Flow diagram of cuckoo search (CS)

New solutions $\boldsymbol{x}^{(t+1)}$ for cuckoo i can be generated by using a Lévy flight as stated in (4). Symbol Lévy (λ) in (4) represents a Lévy flight providing random walk with random step drawn from a Lévy distribution having an infinite variance with an infinite mean as expressed in (5). In another way, the step length s of cuckoo flight can be calculated by (6), where u and v are drawn from normal distribution as stated in (7). Standard deviations of u and v are also expressed in (8), where Γ is the standard Gamma function.

$$\boldsymbol{x}_i^{(t+1)} = \boldsymbol{x}_i^{(t)} + \alpha \oplus \text{Lévy}(\lambda) \quad \textbf{(4)}$$

$$\text{Lévy} \approx u = t^{-\lambda}, \quad (1 < \lambda \le 3) \quad \textbf{(5)}$$

$$s = \frac{u}{|v|^{1/\beta}} \quad \textbf{(6)}$$

$$\left.\begin{array}{l} u \approx N(0.\sigma_u^2) \\ v \approx N(0.\sigma_v^2) \end{array}\right\} \quad \textbf{(7)}$$

$$\begin{array}{l} \sigma_u = \left[\dfrac{\Gamma(1+\beta)\sin(\pi\beta/2)}{\Gamma[(1+\beta)/2]\beta 2^{(\beta-1)/2}} \right]^{1/\beta} \\ \sigma_v = 1 \end{array} \quad \textbf{(8)}$$

4. Results and Discussion

4.1 Parametric Studies

In this work, the CS algorithm was coded by MATLAB for model order reduction of two selected cases of higher-order LTI dynamic systems collected from literatures. Parameter setting of the CS is studied for all cases. The preliminary parametric study over two selected cases is conducted by varying population size of cuckoo (n) = 5, 10,..., 100, Lévy exponent (β) = 0.5, 1.0, 1.5, 2.0 and discovery probability (p_a) = 0.1, 0.2,…, 0.9. From this study, it was found that the best parameters of CS for both two selected cases of model order reduction applications are n = 30, β = 1.5 and p_a = 0.2. The maximum generation Max_Gen = 100 is then set as the termination criteria (TC). Two collected cases of higher-order LTI dynamic systems are reduced to be the second-order models $R(s)$ performed in (9). The order reduction optimization process is to obtain the appropriate five parameters, i.e., a_0, a_1, b_0, b_1 and b_2 in (9). As mentioned in section 1, standard techniques such as L_∞ optimization technique [5] and Routh approximation [6] can be available for solving the model order reduction problem. However, although such the standard techniques consume less computational time, they cannot provide the best results for general cases [14, 15, 16]. For comparison in this work, the GA and PSO, two of popular population-based metaheuristic optimization techniques, are also conducted to solve the model order reduction problems with the same TC. The model reduction optimization runs of 50 trials of each algorithm (GA, PSO and CS) with different random initial solutions in order to obtain the best solution. The search parameters of GA and PSO are set as the recommendations appeared in [8] and [9], respectively.

$$R(s) = \frac{a_1 s + a_0}{b_2 s^2 + b_1 s + b_0} \quad \textbf{(9)}$$

4.2 Case-I (4th Order System)

The first case is the 4th order system as stated in (10) [14]. Referring to the inequality constraints in (3), the corresponding boundaries of five parameters in this case are set as expressed in (11). A priory set of the boundaries of five parameters is arbitrary determined by trial and error schemes. Because setting the boundaries of search space is one of the basic problems of all metaheuristics, two approaches for setting such the boundaries are then given as follows. For the first approach, the boundaries of search space should be set as wide as possible to cover all feasible solution. This approach is very easy for users, but it will spend very long search time consumed. For the second approach, the boundaries should be narrowly set. This approach will spend very short search time, but the solutions found may be locked by upper or lower bounds. Once this occurred, users need to adjust the boundaries to open new space for solution exploration. In this work, after the trial and error schemes finished, such the boundaries in (11) are properly set for this case.

$$G(s)\big|_{4-th} = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24} \quad \textbf{(10)}$$

$$\begin{aligned} \textit{subject to} \quad & 0.1 \le a_0, b_0 \le 10.0 \\ & 0.1 \le a_1, b_2 \le 5.0 \\ & 5.0 \le b_1 \le 15.0 \end{aligned} \quad \textbf{(11)}$$

Results obtained by GA, PSO and CS are summarized in Table 1, where ISE is the integral-squared error, IAE is the integral-absolute error and MISE is the mean of integral-squared error. The second-order models $R(s)$ reduced by GA, PSO and CS are expressed in (12). Time-domain responses, consisting of step, ramp and impulse responses, and frequency responses (Bode diagrams) of the original 4th order system and the second-order models obtained by GA, PSO and CS are depicted in Figure 3, 4, 5 and 6, respectively. Referring to frequency responses in Figure 6, it was found that the second-order models obtained by GA, PSO and CS can retain the bandwidth of the original 4^{th} order system satisfactory. Pole-zero locations of the original 4^{th} order system and the second-order models obtained by GA, PSO and CS are plotted in Figure 7. Although dominant poles of the second-order models obtained by PSO and CS mismatch with those of the original 4^{th} order system as appeared in Figure 7(c) and Figure 7(d), the responses of the second-order models by PSO and CS still retain the dynamic behavior of the original 4^{th} order system. This is because the proposed method in this work is the response-based (not dominant poles-based). By the response-based, the dominant poles of the reduced second-order models may be matched or mismatched with those of the original system. In addition, the second-order models possess not only poles, but also zeros. Responses of the reduced second-order models are shaped by pole and zero effects. However, the dominant poles-based model order reduction is an alternative approach for the future research as the open ended problem. The best convergent rates of the objective function by the GA, PSO and CS are visualized in Figure 8. From Figure 3 - 6, it was found that second-order models reduced by GA, PSO and CS can retain the dynamic behavior of the original 4th order system satisfactory. However, it was found from Table 1 that the CS provides the least ISE, IAE and MISE. This can be noticed that the CS gives the better second-order model than GA and PSO. The convergent rates in Figure 8 confirm that the optimal solutions (parameters) are successfully found by the CS, and the CS can provide the better solution than GA and PSO.

$$\begin{aligned} R(s)\big|_{GA} &= \frac{2.932s+7.885}{3.885s^2+11.48s+7.885} \\ R(s)\big|_{PSO} &= \frac{5.205s+8.989}{6.608s^2+14.89s+8.989} \\ R(s)\big|_{CS} &= \frac{2.974s+6.519}{3.884s^2+10.02s+6.519} \end{aligned} \quad \textbf{(12)}$$

Table 1 Results of case-I obtained by GA, PSO and CS

Algorithms	Parameters of second-order models					Errors		
	a_0	a_1	b_0	b_1	b_2	ISE	IAE	MISE
GA	7.8852	2.9320	7.8851	11.4802	3.8853	8.6920e-04	0.2029	1.0730e-05
PSO	8.9891	5.2052	8.9891	14.8903	6.6082	8.6607e-04	0.1592	1.0688e-05
CS	6.5193	2.9744	6.5193	10.0217	3.8844	7.3933e-04	0.1431	9.1288e-06

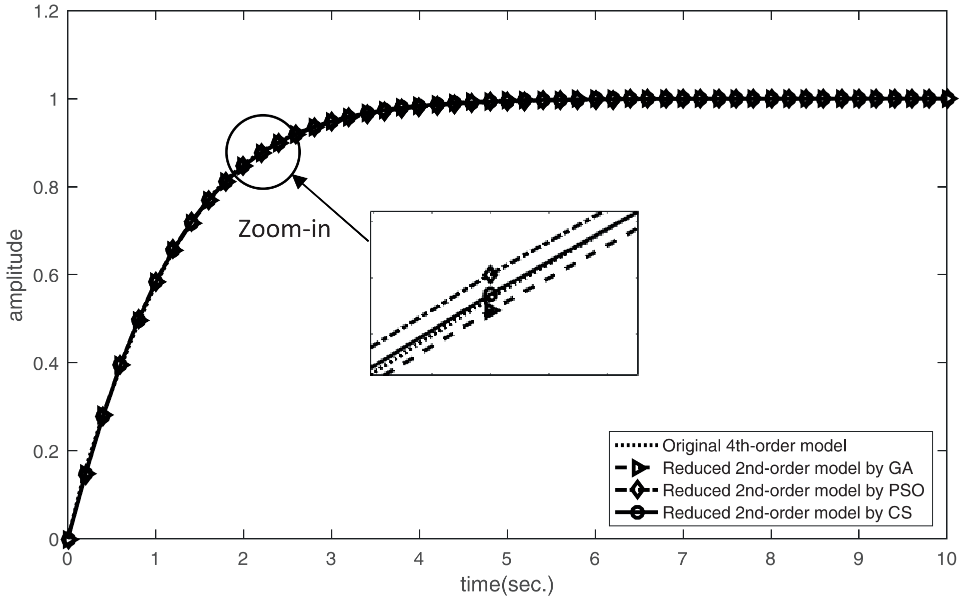


Figure 3 Step responses of original 4^{th} order system and reduced 2^{nd} order models

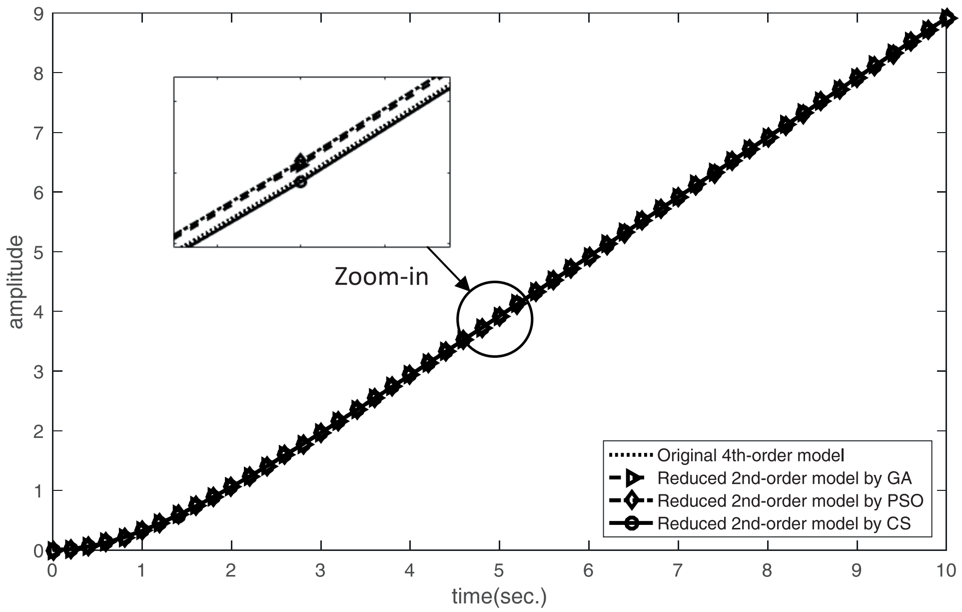


Figure 4 Ramp responses of original 4^{th} order system and reduced 2^{nd} order models

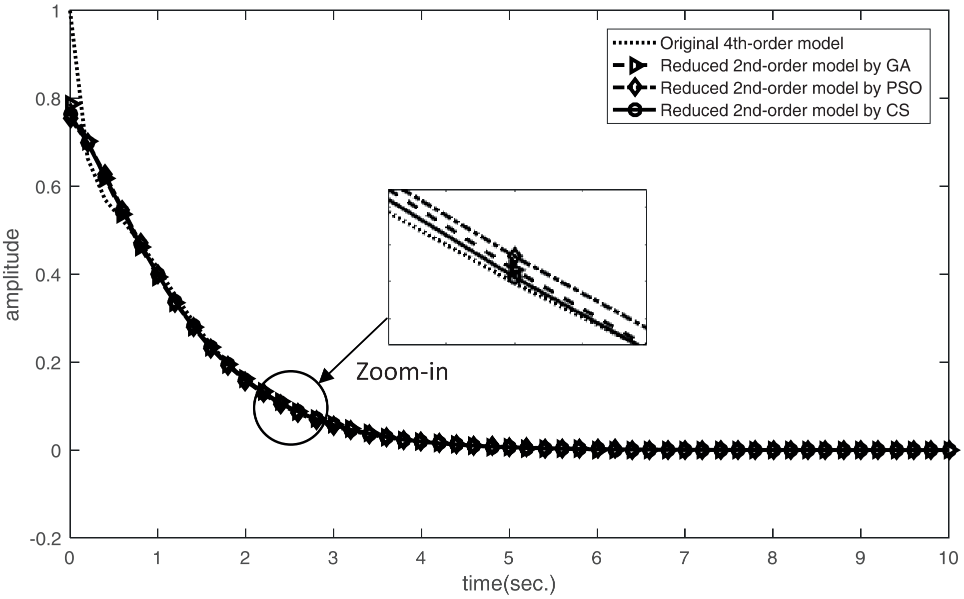
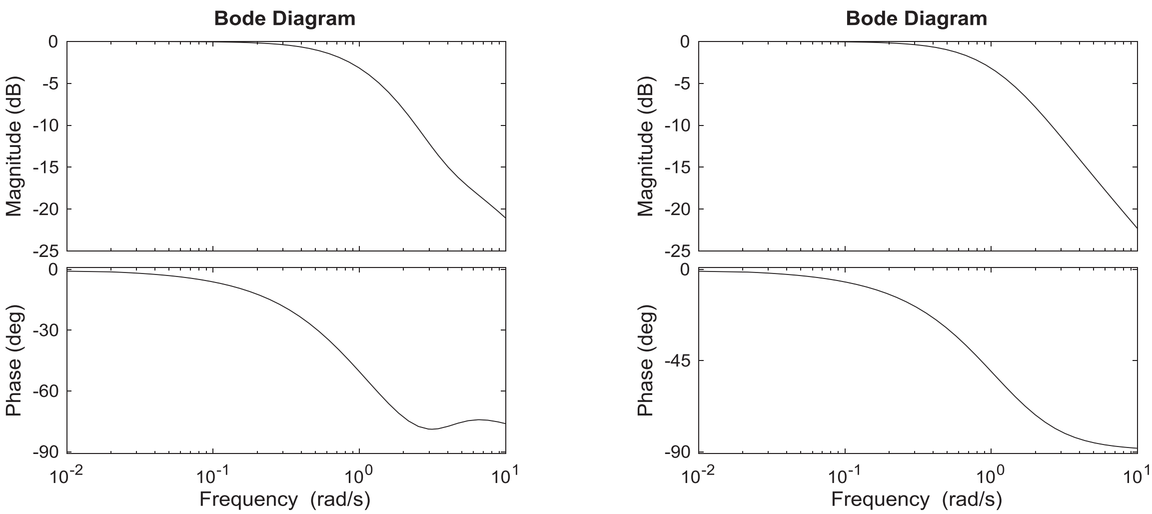


Figure 5 Impulse responses of original 4th order system and reduced 2nd order models



(a) (b)

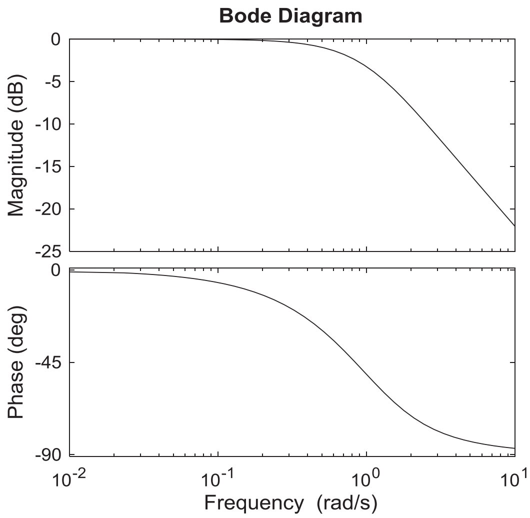
Figure 6 Bode diagrams of original 4th order system and reduced 2nd order models

(a) Bode diagram of original 4th order system

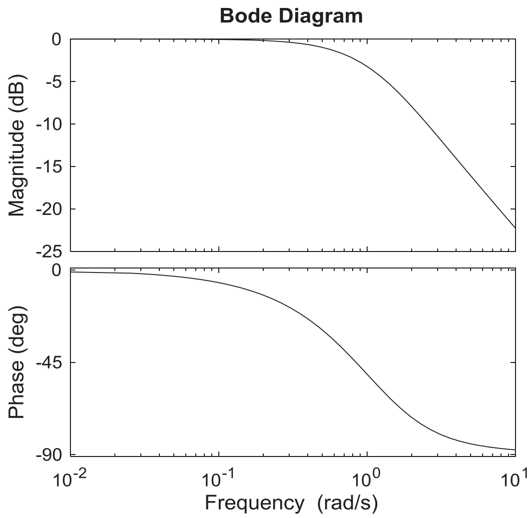
(b) Bode diagram of reduced 2nd order model by GA

(c) Bode diagram of reduced 2nd order model by PSO

(d) Bode diagram of reduced 2nd order model by CS



(c)



(d)

Figure 6 Bode diagrams of original 4^{th} order system and reduced 2^{nd} order models

(a) Bode diagram of original 4^{th} order system

(b) Bode diagram of reduced 2^{nd} order model by GA

(c) Bode diagram of reduced 2^{nd} order model by PSO

(d) Bode diagram of reduced 2^{nd} order model by CS

4.3 Case-II (8^{th} Order System)

For the second case, the 8^{th} order system as expressed in (13) [15, 16] is selected. In this case, b_2=1 is priori set according to the referred works [15, 16]. The boundaries of four parameters in this case are set as stated in (14). Like the case-I, a priory set of the boundaries of four parameters is arbitrary determined by trial and error schemes. After the schemes finished, such the boundaries in (14) are properly set for this case. Results obtained are summarized in Table 2 and the second-order models $R(s)$ reduced by GA, PSO and CS are expressed in (15). Time-domain responses, consisting of step, ramp and impulse responses, and frequency responses (Bode diagrams) of the original 8^{th} order system and the reduced second-order models are depicted in Figure 9, 10, 11 and 12, respectively. Referring to frequency responses in Figure 12, it was found that the second-order models obtained by GA, PSO and CS can retain the bandwidth of the original 8^{th} order system satisfactory. Pole-zero locations of the original 8^{th} order system and the second-order models obtained by GA, PSO and CS are plotted in Figure 13. It was found in this case that dominant poles of the second-order models obtained by GA, PSO and CS match with those of the original 8^{th} order system. The best convergent rates of the objective function by the GA, PSO and CS are visualized in Figure 14. From Figure 9 - 12, all second-order models can retain the characteristics of the original 8^{th} order system satisfactory. However, it was found from Table 2 that the CS provides the least ISE, IAE and MISE. This can be concluded that the CS provides the better second-order model than GA and PSO. Finally, the best convergent rates in Figure 14 assure that the appropriate parameters of the second-order model are completely found by the CS.

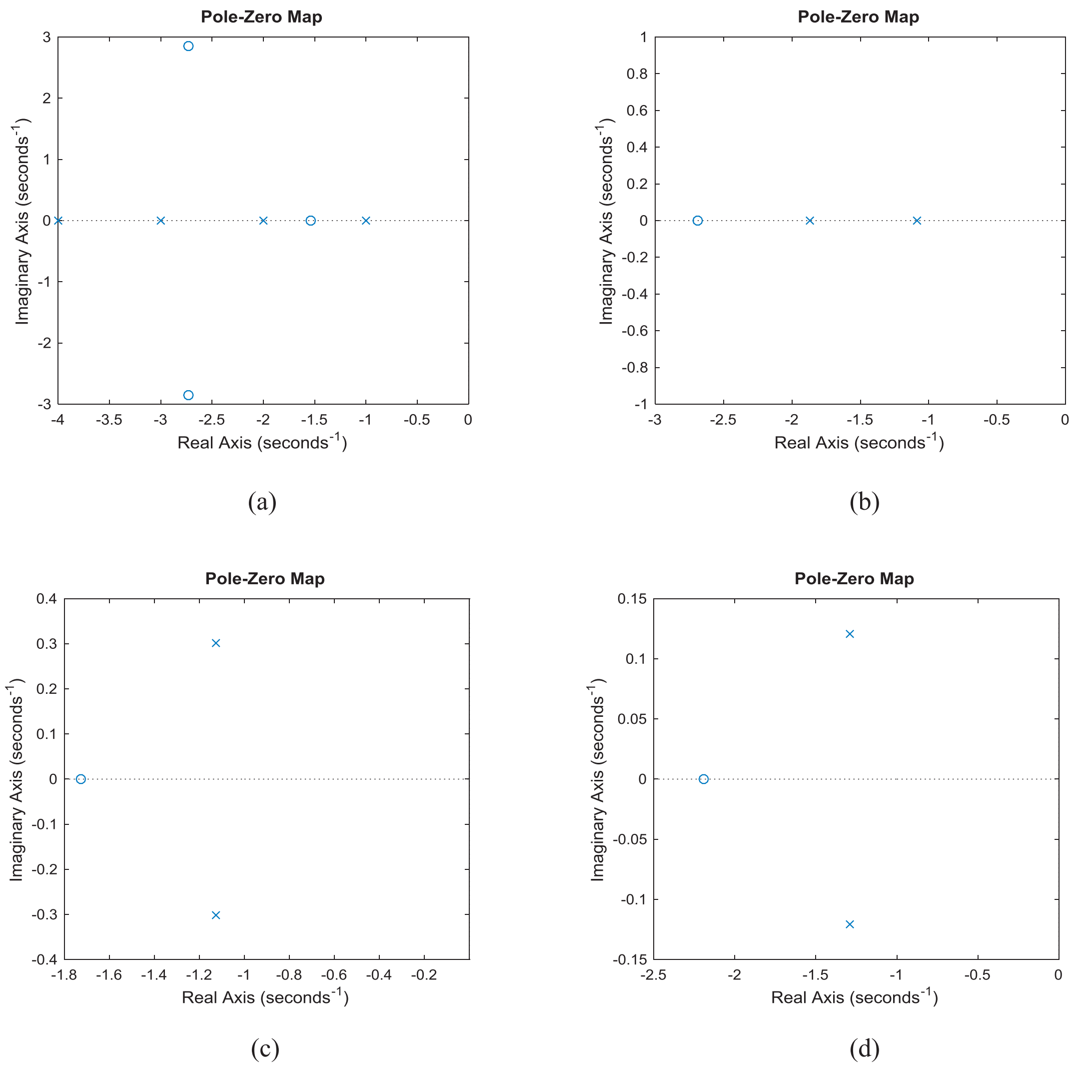


Figure 7 Pole-zero locations of original 4th order system and reduced 2nd order models

(a) Pole-zero locations of original 4th order system

(b) Pole-zero locations of reduced 2nd order model by GA

(c) Pole-zero locations of reduced 2nd order model by PSO

(d) Pole-zero locations of reduced 2nd order model by CS

$$G(s)\big|_{8-th} = \frac{18s^7 + 514s^6 + 5982s^5 + 36380s^4 + 122664s^3 + 222088s^2 + 185760s + 40320}{s^8 + 36s^7 + 546s^6 + 4536s^5 + 22449s^4 + 67284s^3 + 118124s^2 + 109584s + 40320} \quad \textbf{(13)}$$

$$\textit{subject to} \quad \begin{aligned} &0.1 \le a_0, b_0 \le 7.5 \\ &10.0 \le a_1 \le 20.0 \\ &5.0 \le b_1 \le 10.0 \end{aligned} \qquad \textbf{(14)}$$

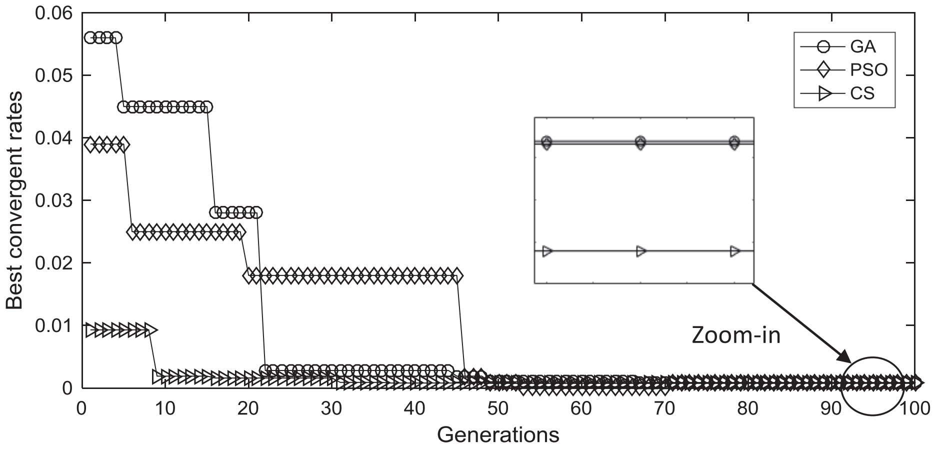


Figure 8 Best convergent rates of the objective function for case-I by GA, PSO and CS

Table 2 Results of case-II obtained by GA, PSO and CS

Algorithms	Parameters of second-order models				Errors		
	a_0	a_1	b_0	b_1	ISE	IAE	MISE
GA	5.0651	17.0124	5.0644	6.8674	0.2572	11.2858	4.1269e-04
PSO	5.0742	17.1012	5.1510	6.9723	0.1092	5.4457	1.1359e-04
CS	5.2593	16.9143	5.2593	6.8686	0.0692	4.0960	6.6833e-05

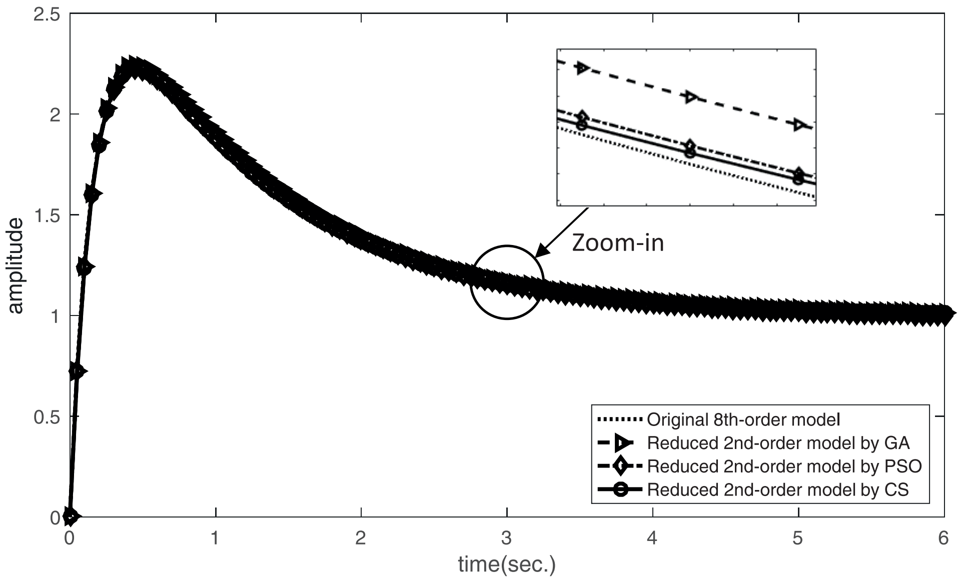


Figure 9 Step responses of original 8th order system and reduced 2nd order models

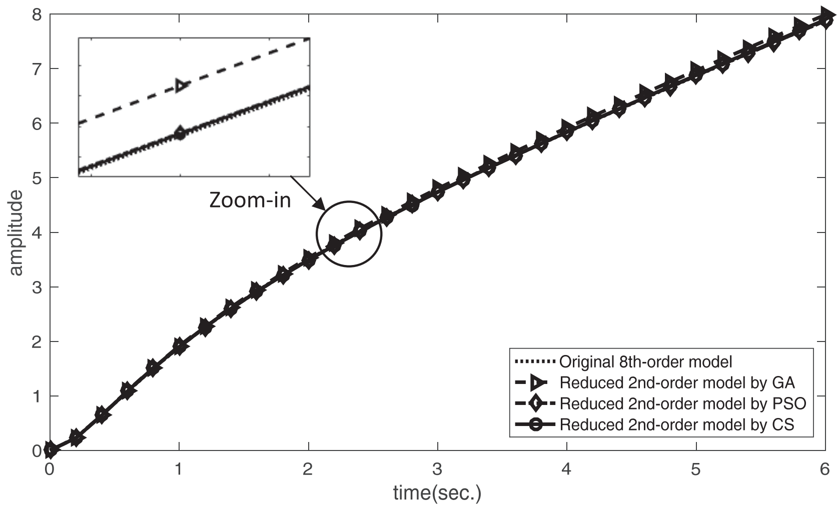


Figure 10 Ramp responses of original 8th order system and reduced 2nd order models

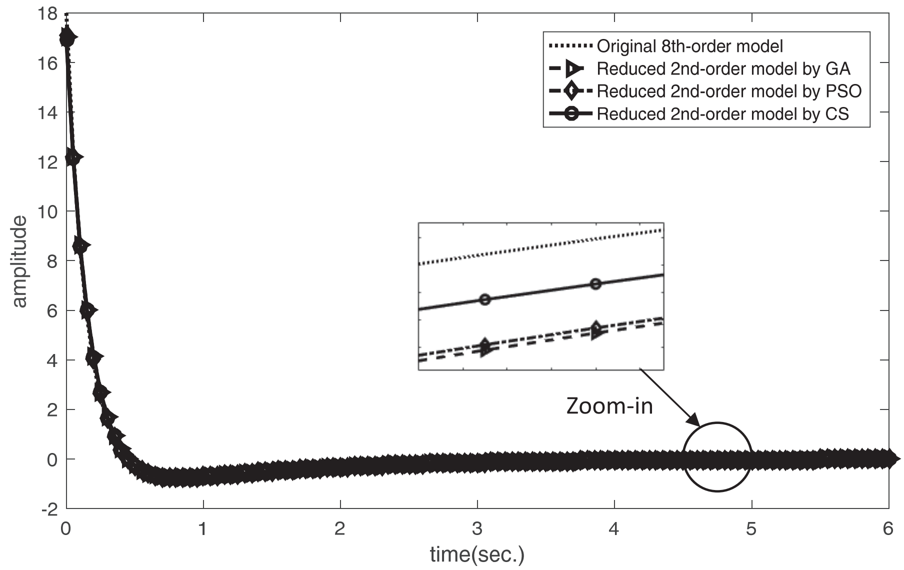


Figure 11 Impulse responses of original 8th order system and reduced 2nd order models

$$R(s)\big|_{GA} = \frac{17.01s + 5.065}{s^2 + 6.867s + 5.064}$$
$$R(s)\big|_{PSO} = \frac{17.10s + 5.074}{s^2 + 6.972s + 5.151} \qquad \textbf{(15)}$$
$$R(s)\big|_{CS} = \frac{16.91s + 5.259}{s^2 + 6.869s + 5.259}$$

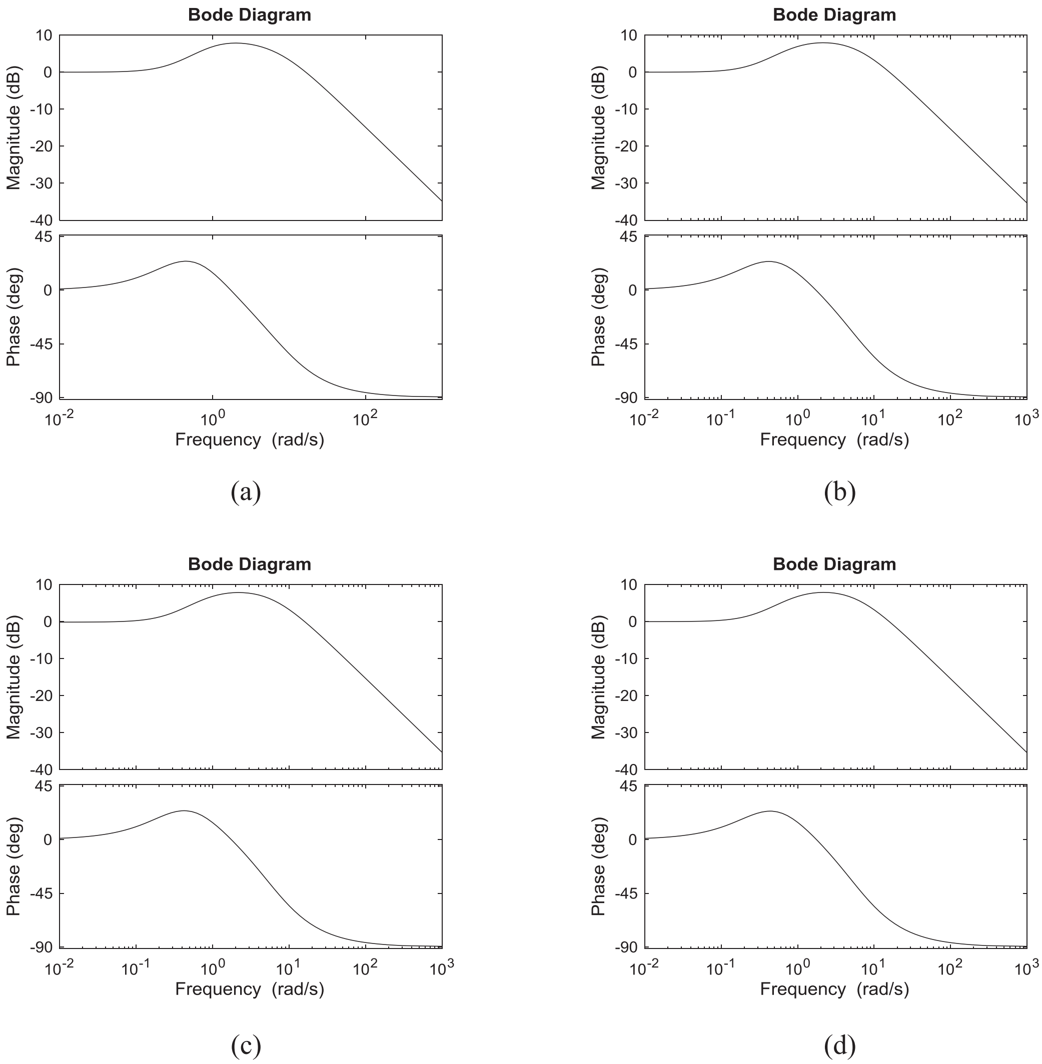


Figure 12 Bode diagrams of original 8th order system and reduced 2nd order models
(a) Bode diagram of original 8th order system
(b) Bode diagram of reduced 2nd order model by GA
(c) Bode diagram of reduced 2nd order model by PSO
(d) Bode diagram of reduced 2nd order model by CS

5. Conclusions

The novel alternative optimization method for model order reduction of LTI dynamic systems by CS has been proposed in this article. The accuracy and efficiency of the proposed method have been performed against two selected higher-order systems and compared with GA and PSO. As simulation results, it was found that the reduced second-order models of two collected higher-order (fourth-order and eighth-order) systems obtained by the CS could retain the system dynamic behavior of original higher-order systems superior to GA and PSO with less IES, IAS and MIES errors. Both time-domain and frequency-domain responses as well as pole-zero locations of the reduced order models have been elaborately shown in order to assure the accuracy and efficiency of the proposed method. This can be concluded that the CS can be efficiently applied to solve the model order reduction problem. In addition, setting the boundaries of search space for any metaheuristics is one of the main drawbacks of modern optimization. Some suggestions for users about setting the boundaries performing the search space have been also provided in this article.

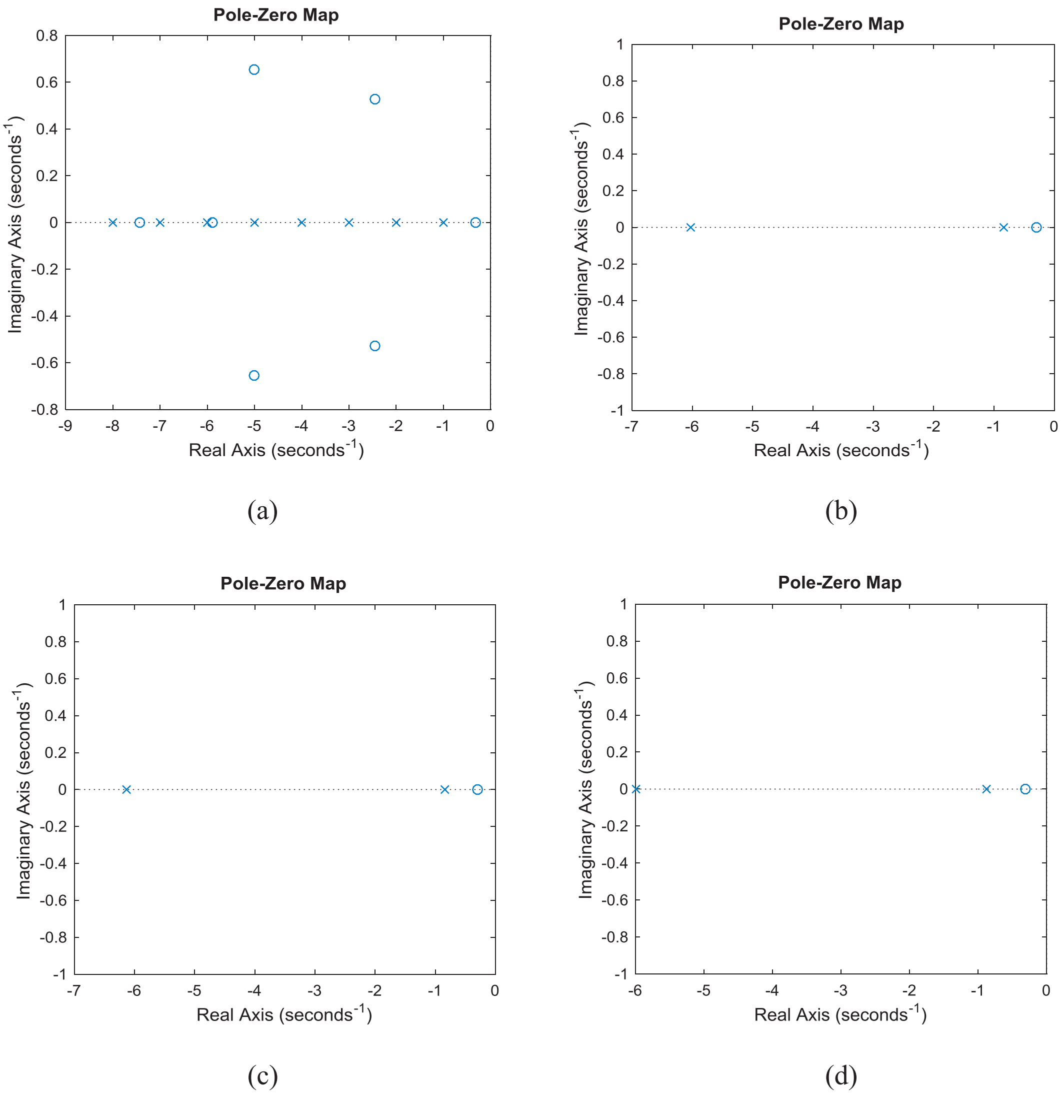


Figure 13 Pole-zero locations of original 8th order system and reduced 2nd order models
(a) Pole-zero locations of original 8th order system
(b) Pole-zero locations of reduced 2nd order model by GA
(c) Pole-zero locations of reduced 2nd order model by PSO
(d) Pole-zero locations of reduced 2nd order model by CS

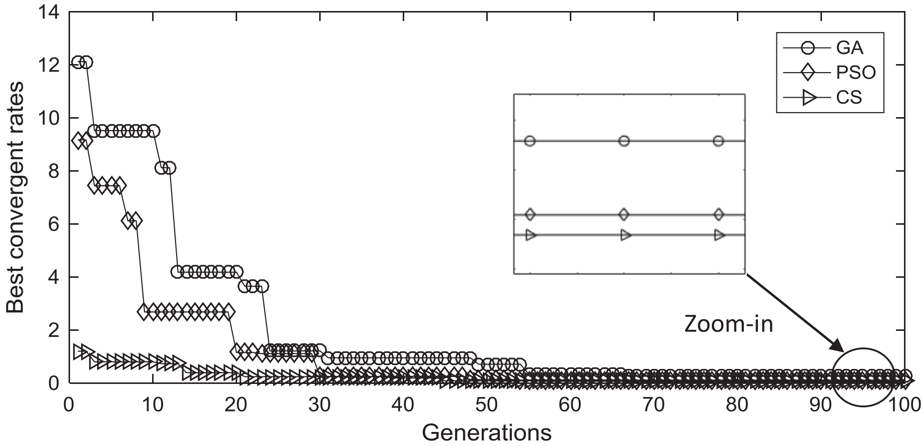


Figure 14 Best convergent rates of the objective function for case-II by GA, PSO and CS

6. References

1. Aoki, M., 1968, "Control of Large-Scale Dynamic System by Aggregation," *IEEE Trans. On Automatic Control*, AC-13, pp. 246-253.

2. Shamash, Y., 1974, "Stable Reduced Order Models using Pade Type Approximations," *IEEE trans. On Automatic Control*, 19, pp. 615-616.

3. Krishnamurthy, V. and Seshadri, V., 1978, "Model Reduction using the Routh Stability Criterion," *IEEE Trans. on Automatic Control*, 23 (4).

4. Sinha, N.K. and Kuszta, B., 1983, Modeling and Identification of Dynamic Systems, Van Nostrand Reinhold, New York, pp. 133-163.

5. Glover, K., 1984, "All Optimal Hankel-Norm Approximations of Linear Multivariable Systems and their L_∞ Error Bounds," *International Journal of Control*, 39 (6), pp. 1115-1193.

6. Hutton, M.F. and Friendland, B., 1999, "Routh Approximations for Reducing Order of Linear Time-Varying Systems," *IEEE Transactions on Automatic Control*, 44 (9), pp. 1782-1787.

7. Glover, F. and Kochenberger, G.A., 2003, Handbook of Metaheuristics, Kluwer Academic Publishers, Dordrecht.

8. Goldberg, D.E., 1989, Genetic Algorithm in Search, Optimization, and Machine Learning, Addison-Wesley Publishing.

9. Kennedy, J. and Eberhart, R., 1995, "Particle Swarm Optimization," *Proceedings of IEEE International Conference on Neural Networks*, 4, pp. 1942-1948.

10. Chutima, P. and Iammi, J., 2003, "Application of Genetic Algorithms in Mixed Model Assembly Line Balancing," *KMUTT Research and Development Journal*, 26 (1), pp. 1-16.

11. Puangdownreong, D., 2007, "Identification of Fourier Coefficients using AI Search Techniques," *KMUTT Research and Development Journal*, 30 (2), pp. 301-314.

12. Puangdownreong, D. and Suwannarongsri, S., 2008, "Application of Eitelberg's Method for PI Controllers via Particle Swarm Optimization," *Proceedings of the 10th International Conference on Control, Automation, Robotics and Vision (ICARCV 2008)*, pp. 566-570.

13. Paksoy, T., Özceylan, E., Pehlivan, N.Y. and Weber, G.W., 2012, "Particle Swarm Optimization Approach for Estimation of Energy Demand of Turkey," *Transaction on Evolutionary algorithm*

and Continuous Optimization, Global Journal of Technology and Optimization, 3, pp. 1-9.

14. Panda, S., Yadav, J.S., Patidar, N.P. and Ardil, C., 2012, "Evolutionary Techniques for Model Order Reduction of Large Scale Linear Systems," *International Journal of Electrical, Computer, Energetic, Electronic and Communication Engineering*, 6 (9), pp. 1105-1111.

15. Nadi, D.A.A., Alsmadi, O.M.K. and Hammour, Z.S.A., 2011, "Reduced Order Modeling of Linear MIMO Systems using Particle Swarm Optimization," *Proceedings of the 7th International Conference on Autonomic and Autonomous Systems (ICAS 2011)*, pp. 62-66.

16. Kumar, S.Y., Ghosh, P.K. and Mukherjee, S., 2011, "Model Order Reduction using Bio-Inspired PSO and BFO Soft-Computing for Comparative Study," *International Journal of Information Systems and Communications*, 1 (1), pp. 43-53.

17. Yang, X.S. and Deb, S., 2009, "Cuckoo Search via Lévy Flights," *Proceedings of the World Congress on Nature and Biologically Inspired Computing (NaBIC 2009)*, pp. 210-214.

18. Yang, X.S. and Deb, S., 2010, "Engineering Optimisation by Cuckoo Search," *International Journal of Mathematical Modelling and Numerical Optimisation*, 1(4), pp. 330-343.

19. Yang, X.S. and Deb, S., 2013, "Multiobjective Cuckoo Search for Design Optimization," *Computers and Operations Research*, 40, pp. 1616-1624.

20. Puangdownreong, D., Kiree, C., Kumpanya, D. and Tunyasrirut, S., 2015, "Application of Cuckoo Search to Design Optimal PID/PI Controllers of BLDC Motor Speed Control System," *Proceedings of the Global Engineering and Applied Science Conference*, Tokyo, Japan, pp. 99-106.

21. Yang, X.S., 2014, "Cuckoo Search and Firefly Algorithm : Overview and Analysis," in Y. Xin-She (Ed.) *Studied in Computational Intelligence : Cuckoo search and Firefly Algorithm Theory and Applications*, Springer, pp. 1-26.